Caroline Lasser, Ilja Klebanov
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All exercises are to be handed in on Thursday, 20.01.2011, before the lecture.

Exercise 1 (Composite Simpson's Rule)
Let $f \in C^4[a,b]$, $I(f) = \int_a^b f(x)dx$.
Based on the error formula for Simpson's rule,

$$Q(f) - I(f) = \frac{1}{90}\left(\frac{b-a}{2}\right)^5 f^{(4)}(\xi) \text{ for some } \xi \in [a,b],$$

where $Q(f) = \frac{1}{6}f(a) + \frac{4}{6}f\left(\frac{a+b}{2}\right) + \frac{1}{6}f(b)$, derive an error formula for the composite Simpson's rule. Is the composite rule exact for cubic polynomials? Justify your answer.

Exercise 2 (Newton-Cotes Formulae)

(a) Integrate the function $f(x) = xe^{-x}cos(2x)$ on the interval $A = [0, 2\pi]$ numerically using the composite midpoint, trapezoidal and Simpson formulae (after deviding A into N intervals of same length).

(b) Plot the error of the three methods in dependence of N in a log-log-scale (the exact result for the integral is given by $I(f) = -\frac{e^{-2\pi}}{25}(10\pi - 3 + 3e^{2\pi}) \approx -0.122122604618968$).

(c) Discuss your results.

Exercise 3 (Monte Carlo Quadrature)
Use the Monte Carlo quadrature method to approximate the integral integral $\int_2^4 xdx$ using N samples:
For each $N = 10, \ldots, 200$ make 10 independent runs and plot their mean values versus N.
Discuss your results.