Caroline Lasser, Ilja Klebanov
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Exercises 1 and 3 are to be handed in on Thursday, 9.12.2010, before the lecture.

Exercise 1 (Bisection method) (∗)

Assume that a continuous function $f : \mathbb{R} \to \mathbb{R}$ has a unique zero in the interval (a, b) and that $f(a)f(b) < 0$ and let a number $n \in \mathbb{N}$ be given. Look at the following pseudo-code:

```
for k = 1, ..., n
   x = (a+b)/2
   display a, b
   if f(x) = 0
      return x
   else if f(x)f(a) < 0
      b = x
   else if f(x)f(b) < 0
      a = x
   end
end
return x
```

(a) What does this algorithm do and why is it called „bisection method“?

(b) Implement this algorithm ($bisec(f, a, b, n)$) and test it for $f(x) = x^2 - 1,\ a = 0,\ b = 3$.

(c) Compare the result with the one from exercise 3 on the 5th exercise sheet (only f_2).

Exercise 2 (Multidimensional Newton's Method)

Consider the (nonlinear) function $F : \mathbb{R}^2 \to \mathbb{R}^2$, $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x^2 - 3xy \\ 5xy \end{pmatrix}$.

(a) Calculate the zeros of F analytically.

(b) Compute the Jocobian matrix $J(x, y)$.

(c) Implement a Matlab function $newtF(x_0, y_0, n)$ which performs and displays n Newton iteration steps for the computation of a zero $(x, y) \in \mathbb{R}^2$ fulfilling $F(x, y) = 0$ starting in (x_0, y_0) and test it for $(x_0, y_0) = (3, 4)$.

Exercise 3 (Aitken's Method) (∗)

Consider a fixed point iteration $x_{n+1} = \varphi(x_n)$ ($x_0 \in \mathbb{R}$ given) and recall Aitken's method:

$$x_{n+1} = x_n - \frac{(\varphi(x_n) - x_n)^2}{\varphi(\varphi(x_n)) - 2\varphi(x_n) + x_n} \, .$$

Use the Δ-operator

$$\begin{aligned} &\Delta(x_n) := x_{n+1} - x_n \\ &\Delta^2(x_n) = \Delta(\Delta(x_n)) = \Delta(x_{n+1} - x_n) = (x_{n+2} - x_{n+1}) - (x_{n+1} - x_n) \end{aligned}$$

to rewrite Aitken's method into a new form (called Aitken's Δ^2-method).