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Exercises 1 and 2 are to be handed in on Tuesday, 30.11.2010, before the lecture.
(or on Monday, 6.12., 10-12 a.m. in office 02.08.059)

Exercise 1 (Making Use of Error Propagation) (∗)
Consider the integrals $\gamma_k = \int_0^1 t^{k-1} e^{1-t} dt,\ k \in \mathbb{N}$.

(a) Show that the numbers γ_k fulfill the *backward-recursion*

$$\gamma_k = \frac{1}{k}(\gamma_{k+1} + 1)\,,\ \ k \in \mathbb{N}.$$

(b) Starting with an index n and an estimated value $\tilde{\gamma}_n = \gamma_n + \Delta\gamma_n$ of γ_n , the upper recursion is applied repeatedly in order to compute $\tilde{\gamma}_m = \gamma_m + \Delta\gamma_m$, $m < n$. What will the error $\Delta\gamma_m$ be (do not work in floating point arithmetics)?

(c) For a given index $m \in \mathbb{N}$ find an index n **and** an estimated value $\tilde{\gamma}_n = \gamma_n + \Delta\gamma_n$, such that (after applying the recursion repeatedly, as described in (b)) the relative error $\frac{\Delta\gamma_m}{\gamma_m}$ of γ_m will be less than 10^{-16}. You may use the fact that $\gamma_k \underset{k\to\infty}{\searrow} 0$.
Implement the resulting algorithm!

(d) Use your program to compute γ_{30} and compare it to the „exact“ result $exp(1) * gammainc(1, 30) * gamma(30)$. Which one is more precise?

Part (d) is meant as an extra exercise for the ambitious students.

Exercise 2 (Fixed Point Iteration) (∗)
Consider the functions $f(x) = cos(x)$ and $g(x) = x^2 - 1$.

(a) Show that f fulfills the assumptions of the theorem concerning the convergence of a fixed point iteration and g does not.

(b) Implement a Matlab function $fixp(\varphi, x_0, n)$ which performs n iteration steps for φ starting in x_0 and illustrates the result graphically. Test your function for $\varphi = f,\ \varphi = g$, respectively (you can use the Matlab function $inline()$ to use functions as parameters, e.g. $f = inline('cos(x)')$).

Exercise 3 (Newton Method)
Write a Matlab function $newt(f, df, x_0, n)$ (as in exercise 2, use the *inline*-function), that performs n iteration steps of the Newton method for the function f, starting in x_0 (df denotes the derivative of f)!
Test your function for $f_1(x) = atan(x),\ f_2(x) = x^2 - 1,\ f_3 = x^3 - 6x^2 + 10x - 2$ and $x_0 = 2$.
For which of these functions does the Newton method converge? Why?