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Exercises 1, 2 and 4 are to be handed in on Thursday, 11.11.2010, before the lecture.

Exercise 1 (Euclidian Norm) ($*$)
Show that the Euclidian norm defined by $\|v\|_2 := \sqrt{\sum_{j=1}^n v_j^2}$ (for $v \in \mathbb{R}^n$) fulfills the properties of a norm, i.e.

1. $\|v\|_2 \geq 0$ for all $v \in \mathbb{R}^n$ and $\|v\|_2 = 0$ if and only if $v_j = 0$ for all $j = 1, \dots, n$,
2. $\|\lambda v\|_2 = |\lambda| \|v\|_2$ (for all $v \in \mathbb{R}^n$, $\lambda \in \mathbb{R}$),
3. $\|v + w\|_2 \leq \|v\|_2 + \|w\|_2$ (for all $v, w \in \mathbb{R}^n$).

Hint: For the third property use the Cauchy-Schwarz inequality presented in the lecture.

Exercise 2 (Quadratic Equations) ($*$)
Consider the quadratic equation $a_0 + a_1 x + a_2 x^2 = 0,\ a_0 \neq 0,\ a_2 \neq 0$. Show that

$$x_1 = \frac{+\sqrt{a_1^2 - 4a_0a_2} - a_1}{2a_2}\ , \quad x_2 = \frac{-\sqrt{a_1^2 - 4a_0a_2} - a_1}{2a_2}$$

solve the upper equation. Prove that these solutions can also be written as

$$x_1 = \frac{2a_0}{-\sqrt{a_1^2 - 4a_0a_2} - a_1}\ , \quad x_2 = \frac{2a_0}{+\sqrt{a_1^2 - 4a_0a_2} - a_1}\ .$$

Which of these formulas would you use to compute x_1, which one to compute x_2?

Hint: Examine the cases $a_1 \geq 0$, $a_1 < 0$ separately and recall that cancellation occurs whenever two similar values are subtracted from one another (consider the cases where $|a_0|$ or $|a_2|$ are very small).

Exercise 3 (Quadratic Equations)
Consider the quadratic polynomial

$$p(x) = (1 + 10^{-161}x)(-1 + 10^{-162}x) = -1 + 9 \cdot 10^{-162}x + 10^{-323}x^2.$$

By construction it has the zeros $x_1 = 10^{162}$, $x_2 = -10^{161}$.
Implement the two formulas for x_1 from exercise 2 and compare the results after subtracting the exact value $x_1 = 10^{162}$.

Exercise 4 (Taylor Polynomial) ($*$)
Consider $f(x) = sin(x)$ and $x_0 = 0$. Calculate the Taylor polynomial $T_n(x)$ of degree

1. $n = 5$,
2. n arbitrary.

(Recall that $sin'(x) = cos(x)$ and $cos'(x) = -sin(x)$.)

Write a Matlab function $taylor(n)$ that plots $f(x) = sin(x)$ and $T_n(x)$ in dependence of n (in one plot) on the interval $[0, 4]$. Test your function for different n.