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Exercises 2 and 4 are to be handed in on Thursday, 04.11.2010, before the lecture.

Exercise 1 (Floating Point Numbers)
Write a Matlab function $floating(b, p, e_{min}, e_{max})$ that calculates and plots the set of floating point numbers

$$\mathbb{F}(b, p, e_{min}, e_{max}) = \left\{\pm m b^{e-p} \mid 0 \le m \le b^p - 1,\ e_{min} \le e \le e_{max}\right\}$$

defined in the lecture.
(Use the Matlab-function sort(a) if you want to sort the vector a.)
Test your function for different parameters b, p, e_{min}, e_{max}.

Exercise 2 (Floating Point Numbers) $(*)$
Let $b = 5,\ p = 1$. Calculate $fl(x)$ for $x = 12,\ x = 2.5$!
You may use the results from exercise 1, but present the result in the form $\pm m b^{e-p}$.
Determine the machine epsilon ϵ_M and compare it two the relative rounding errors of your results. Can you find an $x \in \mathbb{R}$ such that the relative error of $fl(x)$ is at least $\frac{1}{3}\epsilon_M$, $\frac{2}{3}\epsilon_M$ respectively?

Exercise 3 (Fibonacci Numbers)
The Fibonacci numbers f_n $(n \in \mathbb{N})$ are defined by

$$f_n = f_{n-1} + f_{n-2} \text{ (for } n \ge 2),\quad f_0 = 0,\quad f_1 = 1.$$

Write the following Matlab functions:

1. $fib(n)$, the output of which is the list $[f_n, f_{n+1}]$ (e.g. $fib(6) = [8, 13]$).
2. $invfib([a, b], n)$, which inverts the upper recursion n times, in particular $invfib(fib(n), n) = [0, 1]$ (e.g. $invfib([8, 13], 3) = [2, 3]$, $invfib([8, 13], 6) = [0, 1]$).

Test your functions for different input arguments. Calculate $invfib(fib(80), 80)$. Why is it not $[0, 1]$ as it should be?

Exercise 4 (Block Matrices) $(*)$
Let the matrices $A, B \in \mathbb{R}^{n \times n}$ be given in the form

$$A = \begin{pmatrix} A_{n-1} & \alpha \\ \tilde{\alpha} & a \end{pmatrix},\quad B = \begin{pmatrix} B_{n-1} & \beta \\ \tilde{\beta} & b \end{pmatrix},\quad \text{where}$$

$A_{n-1}, B_{n-1} \in \mathbb{R}^{(n-1)\times(n-1)}$ are matrices of lower dimension,
$\alpha, \beta \in \mathbb{R}^{(n-1)\times 1}$ are column vectors,
$\tilde{\alpha}, \tilde{\beta} \in \mathbb{R}^{1\times(n-1)}$ are row vectors,
$a, b \in \mathbb{R}$ are real numbers. Show that the matrix product AB is given by

$$AB = \begin{pmatrix} A_{n-1}B_{n-1} + \alpha\tilde{\beta} & A_{n-1}\beta + \alpha b \\ \tilde{\alpha}B_{n-1} + a\tilde{\beta} & \tilde{\alpha}\beta + ab \end{pmatrix}.$$