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Numerics of Dynamical Systems

Problem Sheet 6

P6.1 Ergodicity of the circle doubling map

(a) Let $f : X \to X$ be a dynamical system with an invariant measure μ. An observable $\varphi \in \mathcal{L}^1(X, \mathcal{B}, \mu)$ is called invariant under f, if $\varphi \circ f = \varphi$ holds μ-almost everywhere.

Show that if only observables φ which are constant μ-a.e. are invariant under f, then μ is ergodic.

(b) Consider now the circle doubling map $f : [0,1] \to [0,1]$, $x \mapsto 2x \bmod 1$. We already know that the Lebesgue measure is invariant with respect to this map. Show that it is also ergodic.

Hint: Compare the Fourier coefficients of φ and $\varphi \circ f$.

P6.2 The Frobenius-Perron operator of the logistic map

We consider the logistic map

$$S : [0,1] \to [0,1], \quad S(x) = 4x(1-x).$$

(a) Write a MATLAB script that computes exactly the discrete Frobenius-Perron operator for a division of $[0,1]$ into N subintervals of equal length.

(b) The left eigenvector corresponding to the eigenvalue 1 gives the approximate invariant density. Plot it and compare it to the exact invariant density

$$h^*(x) = \frac{1}{\pi\sqrt{x(1-x)}}.$$

Try values in the range $N \in [20, 400]$. Can you explain the lack of symmetry?