Prof. Dr. O. Junge, M. Kratzer
Technische Universität München
Winter Semester 2016/17

NUMERICS OF DYNAMICAL SYSTEMS

Problem Sheet 4

P4.1 Convergence speed of the subdivision algorithm

For the linear contraction $f:\mathbb{R}\to\mathbb{R}$, $f(x)=\mu x$, $0<\mu<1$, consider the computation of the relative global attractor of $[-1,1]$ using the subdivision algorithm (with division of intervals at their midpoints).

(a) Compute the number α in the convergence Theorem 2.18.

(b) How many iterations of f are necessary to ensure $\alpha<1$ if (i) $\mu=1/4$, (ii) $\mu=3/4$?

(c) What happens if $\alpha\geq 1$?

P4.2 Continuation algorithm in GAIO

Consider the Lorenz system

$$\begin{aligned}\dot{x} &= \sigma(y-x)\\ \dot{y} &= \rho x - y - xz\\ \dot{z} &= -\beta z + xy\end{aligned}$$

with parameter values $\sigma=10, \rho=28, \beta=\frac{8}{3}$.
The flow map $f:=\phi^{0.1}$ defines a discrete dynamical system. In this exercise, we want to compute the global unstable manifold of the origin.

(a) The file `unstable_manifold.m` on the course website[1] performs the preparation for the continuation algorithm. Complete it by implementing the continuation step and run the script. (Use `help Tree` to get a description of the GAIO methods.)
Optional: To speed up the algorithm, do not recompute $\mathcal{C}_k^{(j+1)}$ from scratch, but find only the new boxes $\mathcal{C}_k^{(j+1)}\setminus\mathcal{C}_k^{(j)}$. Try using flags to do this.

(b) Repeat the computation on the box $Q=[-50,50]^3$ and compare the results.

[1] http://www-m3.ma.tum.de/NumDynSys1617/