Two complementary approaches to event-based control

Zwei komplementäre Zugänge zur ereignisbasierten Regelung

Lars Grüne, Stefan Jerg, Oliver Junge, Daniel Lehmann, Jan Lunze, Florian Müller und Marcus Post

Event-based control aims at reducing the communication effort within a control loop by closing the feedback loop only if an event indicates e.g. a substantial control error. This paper proposes two new methods for event-based control that conjointly guarantee ultimate boundedness of the event-based control loop. The global approach faces the problem of driving a system from its initial state into a desired target set in the state space, whereas the local approach aims at keeping the system state within the target set. Both methods are experimentally evaluated by their application to a thermofluid process.

Das Ziel der ereignisbasierten Regelung ist es, den Kommunikationsaufwand innerhalb eines Regelkreises zu verringern, indem nur dann Daten im Rückführzweig übertragen werden, wenn ein Ereignis z.B. eine große Regelabweichung signalisiert. Dieser Beitrag stellt zwei komplementäre Ansätze zur ereignisbasierten Regelung vor, die auf unterschiedlichen Methoden aufbauen. Dabei hat der globale Ansatz die Aufgabe, ein System ausgehend vom seinem Anfangszustand in ein spezifiziertes Zielgebiet des Zustandsraums zu führen, wohingegen der lokale Ansatz darauf abzielt, den Zustand des Systems unter der Wirkung von Störungen in diesem Zielgebiet zu halten. Es werden experimentelle Ergebnisse der Anwendung beider Ansätze bei der ereignisbasierten Regelung eines thermofluiden Prozesses vorgestellt.

Keywords: Event-based control, state-feedback control, stability analysis, networked control system, set oriented numerics

Schlagwörter: Ereignisbasierte Regelung, Zustandsrückführung, Stabilitätsanalyse, Digital vernetzte Systeme, Mengenorientierte Numerik

1 Introduction

Traditionally, continuous controllers are implemented on digital hardware by sampling continuous-time signals at equidistant instances of time (time-driven sampling). One of the main reasons for applying this approach is a well established systems theory for analysis and design of discrete-time systems [5]. However, this triggering scheme wastes computing and communication resources during time intervals where the system variables do not change significantly, e.g. in steady state.

Event-based control is a means to reduce the computing utilization and the communication between the sensors, the controller and the actuators in a control loop by invoking a communication among these components only after an event has indicated that the control error has exceeded a tolerance bound. The activity of the controller is restricted to time intervals in which the controller inevitably must act in order to guarantee desired specifications of the control loop. The potential of this strategy related to reducing resource utilization has been shown in [2] by simulation, where a conventional discrete-time PID controller was implemented in an event-based way.

Until now only few results on the analysis of event-based control have been published. The heterogeneous terminology describes the triggering mechanism as event-based sampling [3], event-driven sampling [11], Lebesgue sampling [4], deadband sampling [17], send-on-delta sampling [19], level-crossing sampling [14], state-triggered sampling [18] and self-triggered sampling [1].

One of the crucial obstacles for applying event-based sampling at a larger scale is the absence of a fundamental and comprehensive theory. To fill this gap, most work in the field of event-based control is currently concerned with stability analysis and the stabilization of event-based controlled systems with linear and nonlinear dynamics which is also the topic of this paper.

The aim of this contribution is to propose two complementary approaches to event-based control which differ with respect to their control goal and their mathematical background. By conjointly applying these approaches, ultimate boundedness of the event-based closed-loop system can be guaranteed, where the control problem is subdivided into a global and a local control task, each of which is tackled by a distinct approach.

2 Event-based control

2.1 Basic principle

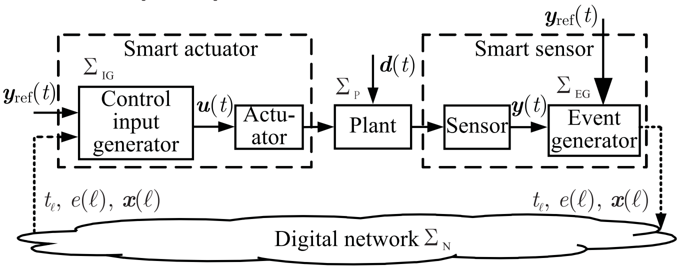

Figure 1: Event-based control loop - continuous-time case

The structure of an event-based control loop is depicted in Fig. 1, where the closed-loop system consists of

- a plant Σ_{P} with actuators and sensors,
- an event generator Σ_{EG}, and
- a control input generator Σ_{IG}.

The necessity of implementing the loop over a digital network Σ_{N} is motivated by the necessity to connect physically far apart sensors and actuators in a control loop. The actuators and sensors are referred to as smart actuators and smart sensors, respectively, which are components with built-in processing units [13]. It is assumed that the processing power of both components does not impose any restrictions on the computational complexity of the control input generator and event generator included.

In contrast to a discrete-time control loop, which is closed at equidistant time steps, the event-based control loop is closed only after events indicate a large control error. The event generator has the task to determine the event times t_ℓ, to generate the corresponding event $e(\ell)$, and to transmit further information such as the state $\boldsymbol{x}(\ell)$, where ℓ denotes the discrete index of events.

The control input generator Σ_{IG} incorporates the controller function and determines the input signal $\boldsymbol{u}(t)$ for the time interval $t \in [t_\ell, t_{\ell+1})$ between two consecutive events in dependence upon the information obtained at time t_ℓ.

Event-based control schemes in general and the particular structure shown in Fig. 1 raise some general questions, which will be discussed in this paper:

- How should events be defined and how should the time instants t_ℓ $(\ell = 0, 1, ...)$ be determined?
- Which information must be communicated?
- How should the control input generator be designed such that ultimate boundedness of the closed-loop system can be guaranteed?

2.2 System description

Since the event generator and the control input generator have to be implemented on smart sensors and actuators by means of digital hardware, a discrete-time plant model

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{f}(\boldsymbol{x}(k), \boldsymbol{u}(k), \boldsymbol{d}(k)), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{y}(k) &= \boldsymbol{g}(\boldsymbol{x}(k), \boldsymbol{u}(k), \boldsymbol{d}(k)), \end{aligned} \tag{1}$$

is considered which represents the continuous plant together with a zero-order hold and a sampler (Fig. 2). The system is sampled at equidistant time instances with sampling period T_{s}.

Throughout this paper a scalar is denoted by an italic ($x \in \mathbb{R}$), a vector by a boldface letter ($\boldsymbol{x} \in \mathbb{R}^n$) and a signal at discrete time k by $\boldsymbol{x}(k)$, where $\boldsymbol{x}(k) \in \mathcal{X} \subseteq \mathbb{R}^n$ denotes the continuous state with the initial value $\boldsymbol{x}_0$. $\boldsymbol{u}(k) \in \mathcal{U} \subseteq \mathbb{R}^m$ and $\boldsymbol{y}(k) \in \mathcal{Y} \subseteq \mathbb{R}^r$ are the exogenous input or measured output, respectively. The set of admissible disturbances is given by $\mathcal{D} = \{\boldsymbol{d} \mid \|\boldsymbol{d}\| < d_{\max}\} \subseteq \mathbb{R}^l$. An event is denoted by $e \in \mathcal{E} \subseteq \mathbb{N}$.

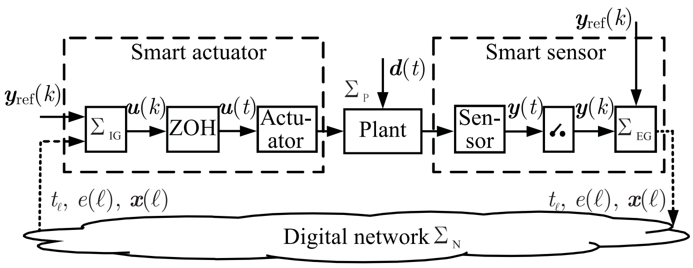

Figure 2: Event-based control loop - discrete-time case

In contrast to the continuous event-based control loop introduced in Sec. 2.1, the discrete-time system induces a significant change: the time when an event occurs can not be precisely determined because of the sampler. This is illustrated in Fig. 3, where the arrow "event" indicates the time instant at which information is communicated. An event should be generated if the state trajectory $x(t)$ (dotted line) reaches the threshold $\overline{x}$. As opposed to the continuous case where the threshold condition $x(t) = \overline{x}$ can exactly be maintained, an event is generated at the subsequent discrete time step. The discrete time k corresponding to the event time t_ℓ is denoted by k_ℓ.

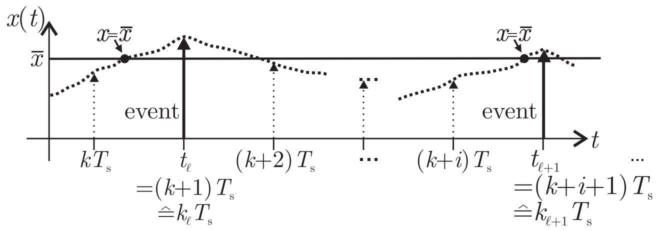

Figure 3: Event-based sampling - discrete-time case

2.3 Control problem

Before the control problem is stated, ultimate boundedness is defined as follows:

Definition 1. ***(Ultimate boundedness [11])*** *A discrete-time system* (1) *is called ultimately bounded (UB) to the set* Ω_d*, if for each* $\boldsymbol{x_0} \in \mathbb{R}^n$ *there exists a time* $T(\boldsymbol{x_0}) > 0$ *such that any state trajectory of the system with initial condition* $\boldsymbol{x}_0$ *(and any admissible realization of the disturbance* $\boldsymbol{d}(k) \in \mathcal{D}$*) satisfies* $\boldsymbol{x}(k) \in \Omega_\mathrm{d}\ \forall k \geq T(\boldsymbol{x}_0)$.

Problem 1. *Given are a plant* Σ_P (1) *and a target set* Ω_d*. Find a controller* $\boldsymbol{K}: \mathcal{E} \to \mathcal{U}$ *or* $\boldsymbol{K}: \mathcal{Y} \to \mathcal{U}$*, respectively, such that the closed-loop system is ultimately bounded to* Ω_d*.*

The solution to Problem 1 should be based on as few communications as possible between the smart sensor and the smart actuator.

2.4 A global and a local approach

This paper proposes two complementary approaches to event-based control based on a decomposition of Problem 1 into two subproblems (Fig 4).

1. Drive the state $\boldsymbol{x}(k)$ of the plant into a region Ω_d by means of a **global event-based approach**.
2. Maintain the state $\boldsymbol{x}(k)$ in the set Ω_d in spite of the presence of exogenous disturbances using a **local event-based approach**.

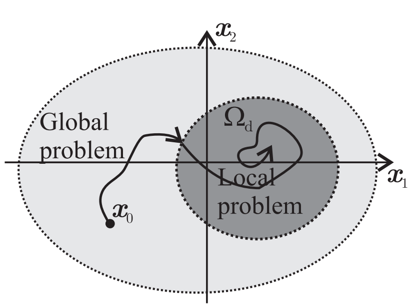

Figure 4: Subdivision into a global and local control problem

Global approach - nonlinear case. In Sec. 3 an approach to optimally steering a nonlinear control system to a desired target set Ω_d is proposed. The state feedback law designed uses quantized information $[\boldsymbol{x}(k)]$ about the state $\boldsymbol{x}(k)$. This quantization naturally partitions the state space into a grid of (coarse) boxes in which the control input $\boldsymbol{u}(k)$ has to be constant. The event-based character lies in updating the input signal only after a crossing of a box boundary occurs, which is detected by the event generator. Practically, the control input generator is implemented by a look-up table which can be computed offline. A major aspect of this approach is that no estimates on state uncertainties have to be carried out while the system operates because the quantized information $[\boldsymbol{x}(k)]$ is sufficient for determining the input $\boldsymbol{u}(k)$.

Local approach - linear case. The event-based scheme introduced in Sec. 4 is based on state-feedback considerations and aims at maintaining the state in the set Ω_d. In this approach the event generator determines the event times t_ℓ by comparing the state of the plant with that of a corresponding discrete-time state-feedback loop. Here, a linearized model of the plant is used.

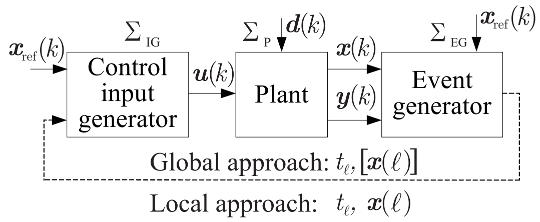

Figure 5: Event-based control loop

For the following investigations the control loop (Fig. 2) is simplified as shown in Fig. 5. The crucial difference among both approaches lies in the implementation of the event generator and the control input generator as well as in the information communicated over the feedback channel.

3 The global approach

3.1 Main idea

In this section the problem of optimally controlling a nonlinear control system to a desired target set by means of a quantized state-feedback law is considered. We assume that

- for the evaluation of the feedback law only (coarsely) quantized measurements are available, and
- only the region containing the initial state and the subsequent crossings of thresholds – the events – are transmitted to the feedback controller.

For the calculation of the feedback law we present a set oriented approach which has been previously proposed for sampled-data systems (see [7, 9, 10, 12]) and transfer the ideas to the event-based setting. Then we address the question of how to solve the problem on very coarsely quantized state spaces by incorporating past data. The key aspect here is that the information on past events reduces the uncertainty about the actual state of the system.

We denote a finite partition of $\mathcal{X}$ by P and the induced *correlation function* $\rho : \mathcal{X} \to P$ is defined as $\rho(\boldsymbol{x}) = \mathcal{P}_i$ for $\boldsymbol{x} \in \mathcal{P}_i$, i.e. the function returns the set $\mathcal{P}_i$ of the partition P in which the state $\boldsymbol{x}$ is located. Furthermore the power set of $\mathcal{X}$ is denoted by $2^{\mathcal{X}}$, the set of sequences by $(2^{\mathcal{X}})^{\mathbb{N}}$ and the set of all control sequences $\underline{\boldsymbol{u}}$ by $\mathcal{U}^{\mathbb{N}}$.

3.2 The event-based model

Starting from a discrete-time model

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k), \boldsymbol{u}(k)),\ k = 0, 1, \dots,$$

$(\boldsymbol{x}(k) \in \mathcal{X}, \boldsymbol{u}(k) \in \mathcal{U})$, which we interpret as a continuous-time sampled-data system describing the plant Σ_{P}, we develop a corresponding event-based system which is induced by a partition P of the state space. We consider an *event* to be triggered whenever the state crosses the boundary of a partition element. Particularly, by $e_{i,j}$ we denote the event which corresponds to the state moving from $\mathcal{P}_i$ to $\mathcal{P}_j$.

Firstly, we define the r-th iterate $\boldsymbol{f}^r(\boldsymbol{x}, \boldsymbol{u})$ for $r \in \mathbb{N}_0$ by $\boldsymbol{f}^0(\boldsymbol{x}, \boldsymbol{u}) := \boldsymbol{x}$, $\boldsymbol{f}^{r+1}(\boldsymbol{x}, \boldsymbol{u}) := \boldsymbol{f}(\boldsymbol{f}^r(\boldsymbol{x}, \boldsymbol{u}), \boldsymbol{u})$, and for each $\boldsymbol{x} \in \mathcal{X}$ with $\boldsymbol{x} \in \mathcal{P}_i$ and each $\boldsymbol{u} \in \mathcal{U}$ we define the function $r(\boldsymbol{x}, \boldsymbol{u})$ to be the smallest value $r \in \mathbb{N}$ satisfying

$$\boldsymbol{f}^{r-1}(\boldsymbol{x}, \boldsymbol{u}) \in \mathcal{P}_i,\ \ \boldsymbol{f}^r(\boldsymbol{x}, \boldsymbol{u}) \in \mathcal{P}_j \quad \text{for some } j \neq i\,. \tag{2}$$

In other words, $r(\boldsymbol{x}, \boldsymbol{u})$ is the time when the event $e_{i,j}$ is generated. Using this concept we can define an *event-based control system*

$$\boldsymbol{x}(\ell+1) = \widetilde{\boldsymbol{f}}(\boldsymbol{x}(\ell), \boldsymbol{u}(\ell)),\ \ \ell = 0, 1, \dots, \tag{3}$$

by

$$\widetilde{\boldsymbol{f}}(\boldsymbol{x}(\ell), \boldsymbol{u}(\ell)) := \boldsymbol{f}^{r(\boldsymbol{x}(\ell), \boldsymbol{u}(\ell))}(\boldsymbol{x}(\ell), \boldsymbol{u}(\ell)) \tag{4}$$

with $\widetilde{\boldsymbol{f}} : \mathcal{X} \times \mathcal{U} \longrightarrow \mathcal{X}$. We also define a cost function for the event-based control system (3) by

$$c_1 : \mathcal{X} \times \mathcal{U} \longrightarrow \mathbb{R}_{+,0}, \quad c_1(\boldsymbol{x}, \boldsymbol{u}) := c^{r(\boldsymbol{x}, \boldsymbol{u})}(\boldsymbol{x}, \boldsymbol{u}), \tag{5}$$

with $r(\boldsymbol{x}, \boldsymbol{u})$ from (2), $c : \mathcal{X} \times \mathcal{U} \to \mathbb{R}, c \geq 0$ a continuous running cost and $c^{r(\boldsymbol{x}, \boldsymbol{u})}(\boldsymbol{x}, \boldsymbol{u})$ defined as $c^1(\boldsymbol{x}, \boldsymbol{u}) := c(\boldsymbol{x}, \boldsymbol{u})$ and $c^r(\boldsymbol{x}, \boldsymbol{u}) := c^{r-1}(\boldsymbol{x}, \boldsymbol{u}) + c(\boldsymbol{f}^{r-1}(\boldsymbol{x}, \boldsymbol{u}), \boldsymbol{u})$.

The global optimal control problem we address can be stated as follows:

Problem 2. ***(Global optimal feedback control problem)*** *Given a model of the plant* (3)*, a target set* $\Omega_{\mathrm{d}} \subset \mathcal{X}$*, and the cost functional along a trajectory* $x(l, \boldsymbol{x}_0, \underline{\boldsymbol{u}})$

$$J(\boldsymbol{x}_0, \underline{\boldsymbol{u}}) = \sum_{\ell=0}^{N(\boldsymbol{x}_0, \underline{\boldsymbol{u}})} c_1(x(\ell, \boldsymbol{x}_0, \underline{\boldsymbol{u}}), \boldsymbol{u}(\ell)), \tag{6}$$

where $N(\boldsymbol{x}_0, \underline{\boldsymbol{u}})$ *is the first time when the target set is reached, compute a feedback law which steers the system into* Ω_{d} *and minimizes functional* (6).

3.3 Basic principle – interpreting a discretization as a perturbation

Before we come to the main idea of how to deal with the event-based approach we introduce some general concepts for optimal feedback control. Standard approaches for solving an optimal control problem (Problem 2) utilize the fact that for the so-called *optimal value function* $V(\boldsymbol{x}) := \inf_{\underline{\boldsymbol{u}} \in \mathcal{U}^{\mathbb{N}}} J(\boldsymbol{x}_0, \underline{\boldsymbol{u}})$ the *optimality principle of Bellman* holds, i.e. $V(\boldsymbol{x}) := \inf_{\boldsymbol{u} \in \mathcal{U}} \{c(\boldsymbol{x}, \boldsymbol{u}) + V(f(\boldsymbol{x}, \boldsymbol{u})\}$. The main idea for the event-based approach is to interprete the quantization of states as uncertainties and to adopt the Bellman equation accordingly. Formally, a dynamic game has to be solved.

Dynamic games. Our *dynamic game* under consideration will be related to the event-based model of the plant $\widetilde{\boldsymbol{f}}$. It is defined by the map $F : P \times \mathcal{U} \times \widehat{\mathcal{C}} \to P$, $F(\mathcal{P}, \boldsymbol{u}, \widehat{\gamma}) := \rho(\widetilde{\boldsymbol{f}}(\widehat{\gamma}(\mathcal{P}), \boldsymbol{u}))$ and the cost function $c_2 : P \times \mathcal{U} \to [0, \infty)$; $c_2(\mathcal{P}, \boldsymbol{u}) := \sup_{\boldsymbol{x} \in \mathcal{P}} c_1(\boldsymbol{x}, \boldsymbol{u})$, where P is a partition of $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{U} \subset \mathbb{R}^m$ is a compact set. $\boldsymbol{\gamma} : (2^{\mathcal{X}})^{\mathbb{N}} \times \mathcal{U}^{\mathbb{N}} \longrightarrow \mathcal{X}^{\mathbb{N}}$, $\boldsymbol{\gamma}(\underline{X}, \underline{\boldsymbol{u}}) = (\widehat{\gamma}_0(\mathcal{X}(0), \boldsymbol{u}(0)), \widehat{\gamma}_1(\mathcal{X}(1), \boldsymbol{u}(1)), \dots)$ is a *choice function* as in [7]. The components $\widehat{\gamma}$ of the choice function $\boldsymbol{\gamma}$ choose a state $\boldsymbol{x} \in \mathcal{X}$ depending on the control $\boldsymbol{u}$ and therefore are helpful to model an uncertainty of the state. The set of all choice functions $\boldsymbol{\gamma}$ is denoted by $\mathcal{C}$ and the set of all component functions $\widehat{\gamma}$ by $\widehat{\mathcal{C}}$. Specifying a target set $\Omega_{\mathrm{d}} \subset \mathcal{X}$, the total cost accumulated along a disturbed trajectory can be defined as

$$J(\boldsymbol{x_0}, \underline{\boldsymbol{u}}, \boldsymbol{\gamma}) = \sum\nolimits_{\ell=0}^{N(\boldsymbol{x}_0, \underline{\boldsymbol{u}}, \boldsymbol{\gamma})} c_2(\rho(x(\ell, \rho(\boldsymbol{x}_0), \underline{\boldsymbol{u}}, \boldsymbol{\gamma})), \boldsymbol{u}(\ell))$$

and the *value function* of the game

$$V(\boldsymbol{x}) = \sup_{\boldsymbol{\gamma} \in \mathcal{C}} \inf_{\underline{\boldsymbol{u}} \in \mathcal{U}^{\mathbb{N}}} J(\boldsymbol{x}, \underline{\boldsymbol{u}}, \boldsymbol{\gamma}(\rho(\boldsymbol{x}), \underline{\boldsymbol{u}})) \tag{7}$$

is to be considered. Here, $\mathcal{C}$ consists of all non-anticipating strategies (for more details see [10]). By standard dynamic programming arguments [6] and if c_2 does not depend on $\boldsymbol{\gamma}$, one sees that this function is the unique solution to the optimality principle

$$V(\boldsymbol{x}) = \inf_{\boldsymbol{u} \in \mathcal{U}} \left\{ c_2(\rho(\boldsymbol{x}), \boldsymbol{u}) + \sup_{\widehat{\gamma} \in \widehat{\mathcal{C}}} V(F(\rho(\boldsymbol{x}), \boldsymbol{u}, \widehat{\gamma})) \right\}. \tag{8}$$

The discrete dynamics maps a partition element to a set of subsets $N \subset P$ which together with the cost function c_2 can be interpreted as a hyperedge in a directed weighted hypergraph $(P, \mathcal{E})$ (Fig. 6). The solution of

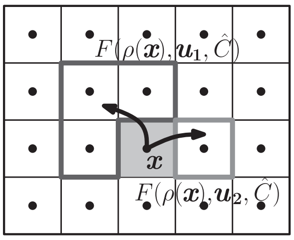

Figure 6: Partition of phase space (grid), images of a partition element correspond to hyperedges in an induced hypergraph.

the discrete game by a modified shortest path algorithm enables us to define the *feedback* $\boldsymbol{u}_P$ as the minimizing argument of (8). This feedback is robust under a perturbation $\mathcal{W}(\boldsymbol{x})$ which is bounded by the elements of the partition, i.e. $\{\boldsymbol{x}\}+\mathcal{W}(\boldsymbol{x}) = \rho(\boldsymbol{x})$. In this sense, the perturbation induces the discretization into partition elements. Summarized, this method guarantees that the application of the optimal control will drive the system into a desired goal region Ω_d even though the controller receives only quantized information. Next, we address the issue of dealing with event histories within the above approach.

3.4 Extension – including past data

Our aim is to stabilize the system on a very coarse quantization of the state space which motivates the use of past data. We illustrate the idea for a history length of one event but the idea also applies to arbitrary length. For this purpose we introduce a new set valued state $\mathcal{Z}(\ell) = (\mathcal{Z}_1(\ell), \mathcal{Z}_2(\ell))^T \in P_2 := (P \cup \{\delta\}) \times P$, where $\mathcal{Z}_1$ represents the state of the previous step and $\mathcal{Z}_2$ of the actual one. As dynamic game we define $F_2 : P_2 \times \mathcal{U} \times \mathcal{C} \to P_2$, $\mathcal{Z}(\ell+1) = F_2(\mathcal{Z}(\ell), \boldsymbol{u}(\ell), \widehat{\gamma}_\ell)$,

$$F_2(\mathcal{Z}(\ell), \boldsymbol{u}(\ell), \widehat{\gamma}_\ell) := \begin{pmatrix} \mathcal{Z}_2(\ell) \\ F(X(\mathcal{Z}(\ell)), \boldsymbol{u}(\ell), \widehat{\gamma}_\ell) \end{pmatrix} \quad (9)$$

with F from Section 3.3 and

$$X(\mathcal{Z}) := \begin{cases} \mathcal{Z}_2, & \text{if } \mathcal{Z}_1 = \delta \\ \bigcup\limits_{\boldsymbol{x} \in \mathcal{Z}_1 \boldsymbol{u} \in \mathcal{U}} \boldsymbol{f}^{r(\boldsymbol{x},\boldsymbol{u})}(\boldsymbol{x}, \boldsymbol{u}) \ \cap \ \mathcal{Z}_2, & \text{else} \end{cases} \quad (10)$$

with $r(\boldsymbol{x}, \boldsymbol{u})$ from (2). Here the symbol δ represents the "undefined" region, which appears when the system starts at time t_0. For the cost function $\widetilde{c}_2 : P_2 \times \mathcal{U} \to [0, \infty)$; $\widetilde{c}_2(\mathcal{Z}, \boldsymbol{u}) := \sup_{\boldsymbol{x} \in \mathcal{X}(\mathcal{Z})} c_1(\boldsymbol{x}, \boldsymbol{u})$ we define a value function V_2 analogously to Section 3.3.

In the worst-case considerations in Section 3.3 we know only that the state is located in a specific partition element (e.g. $\mathcal{P}_i$). With the additional information of the preceding partition element (e.g. $\mathcal{P}_j$) the set of possible states shrinks since the state has to be contained in $X((\mathcal{P}_j, \mathcal{P}_i)) \subset \mathcal{P}_i$. By the resulting reduction of uncertainty the supremums in $\widetilde{c}_2$ and in the optimality principle are taken over a smaller set. This is the main reason why the values of V_2 are better than that of V (for more details and a proof see [8]). Typically, with the better value function one can stabilize a system by means of a coarser quantization.

3.5 Description of the components

Communication structure. The event-based control loop uses the communication structure shown in Fig. 5. At event time t_ℓ ($\ell = 0, 1, \dots$) the quantization of the current state $[\boldsymbol{x}(k_\ell)]$ and its timestamp is transferred to the control input generator. In the case of event histories of length $\ell > 1$, the control input generator additionally contains a storage element.

Control input generator. The control input is generated as a simple zero-order hold which is constant between two successive events, i.e. in the time interval $[t_\ell, t_{\ell+1})$. Practically, the control input generator is implemented by a simple look-up table which contains the mapping from events respectively from event series to controls. The combination of plant and control input generator can be described by the following state-space model:

$$\boldsymbol{x}(\ell+1) = \widetilde{\boldsymbol{f}}(\boldsymbol{x}(\ell), \boldsymbol{u}_P([\boldsymbol{x}(\ell)])), \quad \ell = 0, 1, \dots$$

For a controller dependent on event histories of length N the corresponding equation changes to $\boldsymbol{x}(\ell+1) = \widetilde{\boldsymbol{f}}(\boldsymbol{x}(\ell), \boldsymbol{u}_P([\boldsymbol{x}(\ell)], [\boldsymbol{x}(\ell-1)], \dots, [\boldsymbol{x}(\ell-N+1)]))$.

Event generator. An event $e_{i,j}$ is generated whenever the boundary of two partition elements P_i and P_j is crossed: $\boldsymbol{x}(\ell) \in P_i$ and $\boldsymbol{x}(\ell+1) \in P_j$ with $i \neq j$. In other words, as long as the controlled trajectory stays within a partition element no communication between the plant and the control input generator takes place.

4 The local approach

4.1 Main idea

This section introduces a local approach which keeps the state $\boldsymbol{x}(k)$ in the target set Ω_d in spite of the presence of exogenous disturbances. The plant (1) is assumed to be asymptotically stable and represented in the surrounding Ω_d of the equilibrium state by the linear state-space model

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k) + \boldsymbol{E}\boldsymbol{d}(k), \ \boldsymbol{x}(0) = \boldsymbol{x}_0, \quad (11)$$

where $\boldsymbol{A} \in \mathbb{R}^{n \times n}$, $\boldsymbol{B} \in \mathbb{R}^{n \times m}$ and $\boldsymbol{E} \in \mathbb{R}^{n \times l}$ are real matrices. The discrete-time state-feedback loop is given by

$$\boldsymbol{x}_\mathrm{SF}(k+1) = \overline{\boldsymbol{A}}\boldsymbol{x}_\mathrm{SF}(k) + \boldsymbol{E}\boldsymbol{d}(k), \ \boldsymbol{x}_\mathrm{SF}(0) = \boldsymbol{x}_0 \quad (12)$$

with $\overline{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}$, where the state feedback matrix $\boldsymbol{K}$ is assumed to be chosen so that the closed-loop system has a satisfactory behavior, particularly with respect to its disturbance attenuation properties.

The main idea of the local approach is to replace the discrete-time state-feedback loop (12) by an event-based control loop so that its performance matches that of the state-feedback loop as well as possible. Therefore, system (12) is used in the later investigations as a reference system.

Definition 2. ***(Robust positive invariance [11])*** *The set $\Omega \subseteq \mathbb{R}^n$ is said to be a robustly positively invariant set for the discrete-time system* (12) *with disturbances in $\mathcal{D}$, if for any $\boldsymbol{x}_{\rm SF} \in \Omega$ and any $\boldsymbol{d} \in \mathcal{D}$ the relation*

$$\overline{\boldsymbol{A}}\boldsymbol{x}_{\rm SF} + \boldsymbol{E}\boldsymbol{d} \in \Omega$$

holds.

Problem 3. *Given the plant model* (11)*, a state feedback $\boldsymbol{K}$, and a set $\Omega_0 \subset \mathbb{R}^n$, where Ω_0 is assumed to be a robustly positively invariant set for the discrete-time state-feedback loop* (12)*. Find an event generator and a control input generator such that the performance of the event-based loop matches that of the state-feedback loop in the sense that there exists a set $\Omega_{\rm d} \supset \Omega_0$ in which the state $\boldsymbol{x}(k)$ of the event-based control loop remains.*

The solution to this problem described below uses the event-based control method, which has been proposed in [15, 16] for continuous-time systems. The motivation to communicate information via the dashed arrows in Fig. 5 may be given by the situation that the disturbance $\boldsymbol{d}$ has an intolerable effect on the plant state $\boldsymbol{x}$.

4.2 Description of the components

The components of the event-based control loop (Fig. 5) are implemented as follows [16].

Communication structure. As shown in Fig. 5, the event time k_ℓ and the plant state $\boldsymbol{x}(k_\ell)$ are sent by the event generator to the control input generator.

Control input generator. In the time interval $k \in [k_\ell, k_{\ell+1})$ between two consecutive events the control input is generated by

$$\boldsymbol{x}_{\rm s}(k+1) = \overline{\boldsymbol{A}}\boldsymbol{x}_{\rm s}(k) + \boldsymbol{E}\widehat{\boldsymbol{d}}_\ell, \quad \boldsymbol{x}_{\rm s}(k_\ell) = \boldsymbol{x}(k_\ell) \quad (13)$$

$$\boldsymbol{u}(k) = -\boldsymbol{K}\boldsymbol{x}_{\rm s}(k), \quad (14)$$

where $\widehat{\boldsymbol{d}}_\ell$ is a constant disturbance estimate. The disturbance estimate $\widehat{\boldsymbol{d}}_\ell$ is determined by the relation

$$\widehat{\boldsymbol{d}}_\ell = \widehat{\boldsymbol{d}}_{\ell-1} + \left(\sum_{j=k_{\ell-1}}^{k_\ell-1} \boldsymbol{A}^{k_\ell-1-j}\boldsymbol{E}\right)^{+} \left(\boldsymbol{x}(k_\ell) - \boldsymbol{x}_{\rm s}^-(k_\ell)\right),$$

where $(.)^+$ denotes a pseudo-inverse and $\boldsymbol{x}_{\rm s}^-(k_\ell)$ is given by

$$\boldsymbol{x}_{\rm s}^-(k_\ell) = \overline{\boldsymbol{A}}^{k_\ell-k_{\ell-1}}\boldsymbol{x}(k_{\ell-1}) + \sum_{j=k_{\ell-1}}^{k_\ell-1} \overline{\boldsymbol{A}}^{k_\ell-1-j}\boldsymbol{E}\widehat{\boldsymbol{d}}_{\ell-1}.$$

Event generator. An event is generated whenever the measured state $\boldsymbol{x}(k)$ leaves the set

$$\Omega(\boldsymbol{x}_{\rm s}) = \{\boldsymbol{x} \mid \|\boldsymbol{x} - \boldsymbol{x}_{\rm s}\| \le \bar{e}\} \quad (15)$$

around the state trajectory $\boldsymbol{x}_{\rm s}(k)$ of the discrete-time state-feedback loop, which is also determined by the event generator by eqn. (13). Hence, if

$$\begin{aligned} \|\boldsymbol{x}(k-1) - \boldsymbol{x}_{\rm s}(k-1)\| &< \bar{e} \quad (16)\\ \|\boldsymbol{x}(k) - \boldsymbol{x}_{\rm s}(k)\| &\ge \bar{e} \end{aligned}$$

holds, an event is generated at time $k := k_{\ell+1}$ and the information $\boldsymbol{x}(k_{\ell+1})$ is communicated to the control input generator.

4.3 Analysis of the event-based control system

For the application of the proposed event-based control scheme the following two properties are important.

Lemma 1. *The state $\boldsymbol{x}(k)$ remains in a bounded surrounding $\Pi(\boldsymbol{x}_{\rm s}(k))$ of the state $\boldsymbol{x}_{\rm s}(k)$ determined by the control input generator generator* (13).

The property follows directly by considering system (11) and eqs. (13)-(16). The $k_{\ell+1}$-th event is generated if the measured state $\boldsymbol{x}(k)$ exceeds the boundary $\partial\Omega = \{\boldsymbol{x} \mid \|\boldsymbol{x} - \boldsymbol{x}_{\rm s}\| = \bar{e}\}$ (eqn. (15)) and the state $\boldsymbol{x}_{\rm s}(k_{\ell+1})$ is made coincide with the state of the plant $\boldsymbol{x}(k_{\ell+1})$. However, condition (16) is satisfied with equality sign in general at a continuous time instant $\widetilde{t}$ between two consecutive sampling instants $\{k_\ell - 1, k_\ell\}$ (cf. Fig. 3). The maximum evolution of $\boldsymbol{x}(t)$ in the time interval $[\widetilde{t}, k_\ell T_{\rm s})$ can be described by an upper bound for the evolution of $\boldsymbol{x}(t)$ in a single discrete-time step (eqs. (11), (14))

$$x_{\max} = \max_{\boldsymbol{x}, \boldsymbol{x}_{\rm s}} \|\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{K}\boldsymbol{x}_{\rm s}\| + \|\boldsymbol{E}\| d_{\max}$$

with $\boldsymbol{x} \in \partial\Omega(\boldsymbol{x}_{\rm s})$. Hence, the plant state $\boldsymbol{x}(k+1)$ remains in the surrounding

$$\Pi(\boldsymbol{x}_{\rm s}(k)) = \{\boldsymbol{x} \mid \|\boldsymbol{x} - \boldsymbol{x}_{\rm s}(k)\| < \bar{e} + x_{\max}\}. \quad (17)$$

Theorem 1. *The difference $\boldsymbol{e}(k) = \boldsymbol{x}(k) - \boldsymbol{x}_{\rm SF}(k)$ between the states of the event-based control loop and the discrete-time state-feedback loop is bounded by*

$$e_{\max} = (\bar{e} + x_{\max}) \cdot \sum_{j=0}^{\infty} \|\overline{\boldsymbol{A}}^j \boldsymbol{B}\boldsymbol{K}\|. \quad (18)$$

Consequently,

$$\Omega_{\rm d} = \{\boldsymbol{x} | \boldsymbol{x} \in \Omega_0 \wedge \|\boldsymbol{x} - \boldsymbol{x}_{\rm SF}\| \le e_{\max}, \forall \boldsymbol{x}_{\rm SF} \in \partial\Omega_0\} \quad (19)$$

is a robustly positively invariant set for the event-based control system.

Proof. For the state difference $\boldsymbol{e}(k) = \boldsymbol{x}(k) - \boldsymbol{x}_{\mathrm{SF}}(k)$ eqs. (12)-(14) yield

$$\boldsymbol{e}(k+1) = \overline{\boldsymbol{A}}\boldsymbol{e}(k) + \boldsymbol{B}\boldsymbol{K}(\boldsymbol{x}(k) - \boldsymbol{x}_{\mathrm{s}}(k)),$$
$$\boldsymbol{e}(0) = \boldsymbol{0}.$$

As the difference $\boldsymbol{x}(k) - \boldsymbol{x}_{\mathrm{s}}(k)$ is bounded according to Lemma 1 and the state-feedback loop is stable, the difference $\boldsymbol{e}(k)$ is bounded as well. Due to eqn. (17) the following relation holds

$$\begin{aligned}\|\boldsymbol{e}(k)\| &\leq \|\sum_{j=0}^{k-1} \overline{\boldsymbol{A}}^{k-1-j}\boldsymbol{B}\boldsymbol{K}(\boldsymbol{x}(j) - \boldsymbol{x}_{\mathrm{s}}(j))\| \\ &\leq \sum_{j=0}^{k-1} \|\overline{\boldsymbol{A}}^{k-1-j}\boldsymbol{B}\boldsymbol{K}\| \cdot \|(\boldsymbol{x}(j) - \boldsymbol{x}_{\mathrm{s}}(j))\| \\ &\leq \sum_{j=0}^{\infty} \|\overline{\boldsymbol{A}}^{j}\boldsymbol{B}\boldsymbol{K}\| \cdot (\overline{e} + x_{\max}) = e_{\max}.\end{aligned}$$

As, by assumption, the set Ω_0 is robustly positively invariant for the state $\boldsymbol{x}_{\mathrm{SF}}$, the set Ω_{d} defined in eqn. (19) is robustly positively invariant for the state $\boldsymbol{x}$ of the event-based control loop. □

5 Application

5.1 Process description

Both event-based strategies have been applied to the thermofluid process depicted in Fig 7. The process

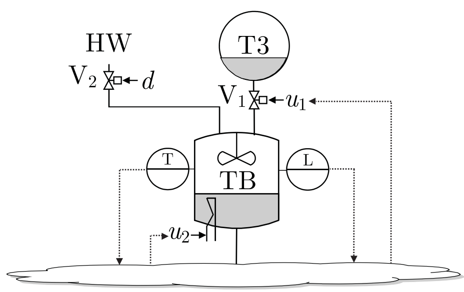

Figure 7: Thermofluid process

consists of the cylindrical batch reactor TB, which is connected with the spherical tank T3 and an additional water supply HW by pipes. The inflow into TB from T3 and HW can be continuously controlled using the valves V_1 and V_2, whereas the outflow only depends on the fluid level in TB. The heating power induced by heating rods can also be continuously adjusted to increase the temperature of the fluid in TB. Moreover, the level l_{TB} and the temperature ϑ_{TB} in TB can be continuously measured.

In this configuration the valve angle $u_1 \in [0, 1]$ of valve V_1 and the power $u_2 \in [0, 6]$ of the heating rods are considered as inputs $\boldsymbol{u}(t) = (u_1(t),\ u_2(t))^{\mathrm{T}}$, whereas the valve angle $d \in [0, 1]$ of valve V_2 is used in order to realize desired disturbance characteristics. With state $\boldsymbol{x}(t) = (l_{\mathrm{TB}}(t), \vartheta_{\mathrm{TB}}(t))^{\mathrm{T}} = (x_1(t), x_2(t))^{\mathrm{T}}$ the nonlinear process Σ_{P} is given as follows (parameters see Tab. 1):

$$\begin{aligned}\dot{x}_1(t) &= \frac{1}{A_{\mathrm{h}}}\big(q_{\mathrm{T3}}(u_1(t)) + q_{\mathrm{HW}}(d(t)) \\ &\quad - K_{\mathrm{A}}\sqrt{2gx_1(t)}\big) \\ \dot{x}_2(t) &= \frac{1}{V(x_1(t))}\Big(q_{\mathrm{T3}}(u_1(t))(\vartheta_{\mathrm{T3}} - x_2(t)) \\ &\quad + q_{\mathrm{HW}}(d(t))(\vartheta_{\mathrm{HW}} - x_2(t)) + \frac{P_{\mathrm{el}}k_{\mathrm{h}}u_2(t)}{\varrho\, c_{\mathrm{p}}}\Big) \\ \boldsymbol{y}(t) &= \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}\end{aligned} \tag{20}$$

$$q_{\mathrm{T3}}(u_1(t)) = \begin{cases} 7\cdot 10^{-6}\cdot(11.1\cdot u_1^2(t) & \\ +13.1\cdot u_1(t) + 0.2) & \text{, for } u_1(t) > 0.2; \\ 0 & \text{, else} \end{cases}$$

$$q_{\mathrm{HW}}(d(t)) = \begin{cases} 1.03\cdot 10^{-4} d(t) & \\ +9.45\cdot 10^{-7} & \text{, for } d(t) > 0.02; \\ 0 & \text{, else} \end{cases}$$

$$V(x_1(t)) = 0.07\cdot x_1(t) - 1.9\cdot 10^{-3} \text{ , for } x_1(t) > 0.26;$$

The unit of the flows q_i is m^3/s and the unit of the volume V of the fluid in TB is m^3.

Table 1: Parameters and constants

Parameter	Value	Meaning
P_{el}	3000 W	Electrical power
k_{h}	0.84 J/(W s)	Heat transfer coefficient
c_{p}	4180 J/(kg K)	Heat capacity of water
g	9.81 $\mathrm{m/s}^2$	Gravitation constant
ϱ	998 $\mathrm{kg/m}^3$	Density of water
ϑ_{U}	293.15 K	Temperature of ambient
ϑ_{HW}, ϑ_{T3}	293.15 K	Temperature of inflows
K_{A}	$1.59\cdot 10^{-5}$ m^3/m	Outflow parameter
A_{h}	$0.07\mathrm{m}^2$	Cross sectional area

For the thermofluid process, Problem 1 is now solved by firstly applying the global event-based scheme (Sec. 5.2) in order to steer the system into a region Ω_{d} around the operating point

$$\begin{aligned}(\overline{x}_1, \overline{x}_2) &= (\bar{l}_{\mathrm{TB}}, \overline{\vartheta}_{\mathrm{TB}}) = (0.349\,\mathrm{m}, 310.56\,\mathrm{K}) \\ (\overline{u}_1, \overline{u}_2) &= (0.34, 1.2).\end{aligned} \tag{21}$$

The results by applying the local approach in order to keep the state of the process subject to exogenous disturbances in a neighborhood around operating point (21) are shown in Sec. 5.3.

5.2 Evaluation of the global approach

For the computation of the feedback by the global approach we restrict the state space to $\mathcal{X} = [0.26,\ 0.45]\,\mathrm{m} \times [293.15,\ 323.15]\,\mathrm{K}$ due to technical limitations. The target set being a neighborhood of the operating point is chosen as $[0.33125\ 0.35500]\,\mathrm{m} \times [308.650\ 312.275]\,\mathrm{K}$ while the underlying cost function is a normalized average of the quadratic distance from the operating point. It is checked every second if an event is triggered within the simulation and the experiment.

As expected from theory, the optimal value function has better values if past data is used and the tank system can be stabilized for a coarser partition. The simulated system can already be stabilized for a grid of 8×8 regular boxes with the use of past data while a finer grid is necessary without using that information. However, for the control of the real process with past data information, a partition of 16×16 boxes is required (Fig. 8 & 9), otherwise the trajectories of experiments would leave the set $\mathcal{X}$ due to unmodeled system reaction times.

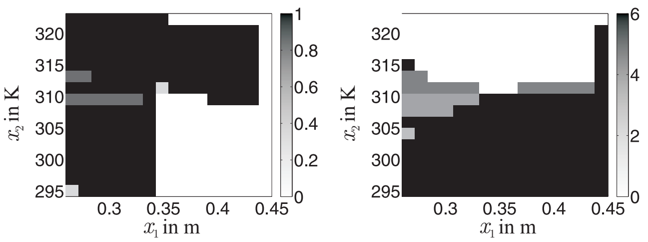

Figure 8: Initial controls $\boldsymbol{u}_1$ (left) and $\boldsymbol{u}_2$ (right) over state-space (16×16 grid) with past data information. Different colors indicate different input values.

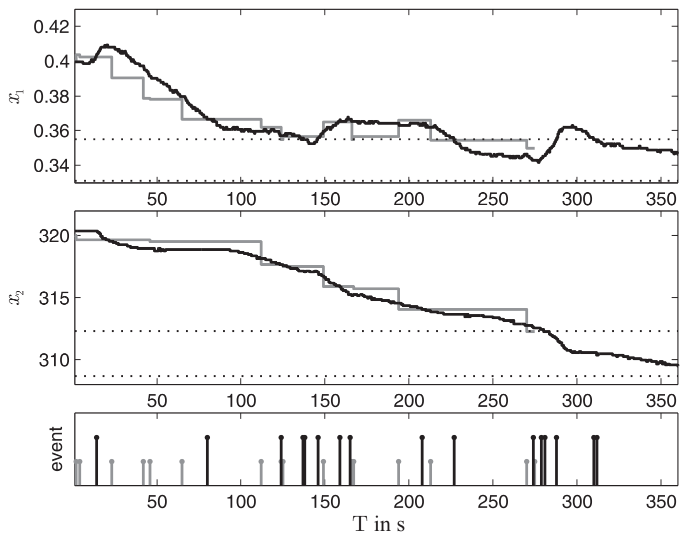

Figure 9: Trajectories for initial state $x(0) = (0.4, 320)$ of the event-based controlled real process (smooth) and simulated process (stairs) using past data on a 16×16 grid. The target region is delimited by the dashed lines.

5.3 Evaluation of the local approach

Based on the results shown in the previous section, the state $\boldsymbol{x}(k)$ is now considered to be in the target set Ω_{d}. This section shows the experimental results obtained by applying the local event-based scheme. For the sampling period $T_{\mathrm{s}} = 1$, the linearized process is described by

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{pmatrix} 0.9992 & 0 \\ -11 \cdot 10^{-11} & 0.9982 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + 10^{-3} \begin{pmatrix} 2.07 & 0 \\ -111.6 & 26.75 \end{pmatrix} \begin{pmatrix} u_1(k) \\ u_2(k) \end{pmatrix} + 10^{-3} \begin{pmatrix} 1.48 \\ -79.78 \end{pmatrix} d(k). \quad (22)$$

The state variables are transformed $x \triangleq x - \overline{x}$, so that the operating point (21) is moved to the origin. In the following investigation the model (22) together with the state feedback matrix

$$\boldsymbol{K} = \begin{pmatrix} 7.64 & -0.18 \\ 16.72 & 0.74 \end{pmatrix} \quad (23)$$

is used by the control input generator $\Sigma_{\mathrm{IG}} = ((13), (14))$ to determine the control input $\boldsymbol{u}(k)$. The event generator Σ_{EG} is implemented using the max-norm

$$\left\| \begin{pmatrix} 100 x_{\Delta,1}(k) \\ x_{\Delta,2}(k) \end{pmatrix} \right\|_\infty = \texttt{max}(100(x_1(k) - x_{\mathrm{s}1}(k)),\ \ x_2(k) - x_{\mathrm{s}2}(k) \geq 2, \quad (24)$$

where $x_{\Delta,1}$ and $x_{\Delta,2}$ are the elements of the vector $\boldsymbol{x}_\Delta$.

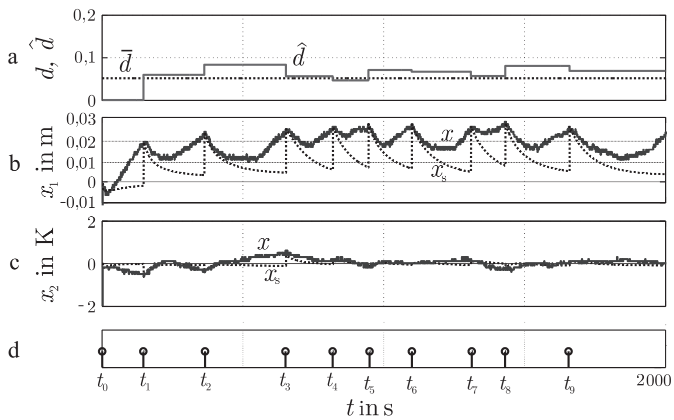

Figure 10: Trajectories of the real process subject to an unknown inflow

In Fig. 10 the resulting trajectories of the continuous process subject to a constant disturbance (Fig. 10a: dotted line) are shown. Figure 10b shows the behavior of the level (x_1: solid line, $x_{\mathrm{s}1}$: dotted line) and Fig. 10c the behavior of the temperature of a fluid in TB. An event takes place at time t_1, where

$$\|\boldsymbol{x}_\Delta(t_1)\|_\infty = 100|x_1(t_1) - x_{\mathrm{s}1}(t_1)| = 2$$

holds (cf. Fig. 10b). At this time instance the disturbance magnitude $\bar{d}$ is estimated. Overall 10 events occur in the time interval $[0, 2000]$ (Fig. 10d) compared to approximately 200 events of a conventional discrete-time control of this process.

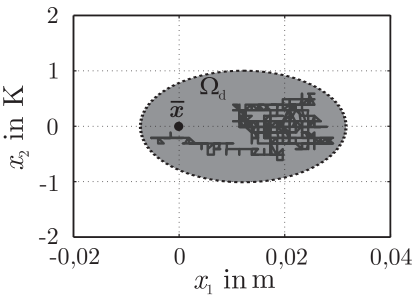

Figure 11: State trajectory in the state-space

Figure 11 shows the state trajectory $\boldsymbol{x}(k)$ in the state-space. The trajectory remains in a region Ω_{d} around operating point (21), where the deviation from the operating point can be explained by the fact that a proportional controller is used. It can be avoided by using controllers with integral action.

6 Conclusions

In this paper two complementary approaches to event-based control are proposed, which are based on state-feedback considerations and a comparable structure of the event-based control loop. The approaches differ with respect to their mathematical background and implementation as well as their control tasks. In order to achieve ultimate boundedness of the closed-loop system, the plant state is driven into a target set with quantized state information by means of the global approach. Communication is invoked only if the boundary of two partition elements is crossed. The problem of keeping the state within the target set is solved by a local event-based scheme, where information is transmitted only if the effect of the disturbance reaches a given threshold. It has been shown that the performance of the event-based control system approximates the behavior of the discrete-time state-feedback loop in the sense that based on a robustly positively invariant set for the discrete-time state-feedback loop a robustly positively invariant set for the event-based control system can be derived. The theoretical results are experimentally evaluated by their application to a thermofluid process.

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Prof. Dr. Lars Grüne Prof. Dr. Lars Grüne is professor for Applied Mathematics at the Mathematical Institute of the University of Bayreuth. Research areas: Mathematical systems and control theory, in particular numerical and optimization based methods for nonlinear systems

Adress: Universität Bayreuth, Lehrstuhl für Angewandte Mathematik, D-95440 Bayreuth,Fax: +49-(0)921-55-5361, E-Mail: lars.gruene@uni-bayreuth.de

Dipl.-Math. Stefan Jerg is PhD student at the Chair of Scientific Computing (M3) at the Technische Universität München. Field of interest: event-based control of non-linear systems with delays.

Adress: Technische Universität München, Zentrum Mathematik, Lehrstuhl Wissenschaftliches Rechnen (M3), Boltzmannstr. 3, D-85748 Garching bei München, Fax: +49-(0)89-289-17985, E-Mail: jerg@ma.tum.de

Prof. Dr. Oliver Junge is professor for Numerics of Complex Systems at the Faculty of Mathematics of the Technische Universität München. Research areas: Numerical analysis, dynamical systems, optimal control.

Adress: Technische Universität München, Zentrum Mathematik, Lehrstuhl Wissenschaftliches Rechnen (M3), Boltzmannstr. 3, D-85748 Garching bei München, Fax: +49-(0)89-289-17985, E-Mail: junge@ma.tum.de

Dipl.-Ing. Daniel Lehmann is PhD student at the Institute of Automation and Computer Control at the Ruhr-University Bochum, Germany. Fields of interest: event-based control, quantized systems, networked control systems.

Adress: Ruhr-Universität Bochum, Lehrstuhl für Automatisierungstechnik und Prozessinformatik, D-44780 Bochum, Fax: +49-(0)234-32-14101, E-Mail: lehmann@atp.rub.de

Prof. Dr.-Ing. Jan Lunze is head of the Institute of Automation and Computer Control at the Ruhr-University Bochum, Germany. Fields of interest: hybrid dynamical systems, process diagnosis, networked control systems.

Adress: Ruhr-Universität Bochum, Lehrstuhl für Automatisierungstechnik und Prozessinformatik, D-44780 Bochum, Fax: +49-(0)234-32-14101, E-Mail: lunze@atp.rub.de

Dipl.-Wirtschaftsmath. Florian Müller is PhD student at the Chair of Applied Mathematics at the University of Bayreuth. Fields of interest: event-based control, quantized systems

Adress: Universität Bayreuth, Lehrstuhl für Angewandte Mathematik, D-95440 Bayreuth, Fax: +49-(0)921-55-5361, E-Mail: florian.mueller@uni-bayreuth.de

Dr. Marcus Post is research assistant at the Chair of Scientific Computing (M3) at the Technische Universität München. Field of interest: event-based control of non-linear systems with delays.

Adress: Technische Universität München, Zentrum Mathematik, Lehrstuhl Wissenschaftliches Rechnen (M3), Boltzmannstr. 3, D-85748 Garching bei München, Fax: +49-(0)89-289-17985, E-Mail: marcus.post@ma.tum.de