Numerical Programming 1 (CSE) 2015

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Worksheet 9

Exercise 1

Given the Newton method

$$x^{k+1} = x^k - \frac{f(x^k)}{f'(x^k)} \qquad \text{with some initial value} \qquad x^0 :$$

- Consider the function $f(x) = x^2$ and prove that the local order of convergence of Newton method is in this case linear.
- Consider the function $f(x) = x^3 - 2x + 2$ and describe the behavior of Newton method when starting from the value $x^0 = 0$.

Exercise 2

Consider the following fixed-point iteration schemes both having $x^0 = 1$ as initial point:

- $$x^{k+1} = \frac{1}{5}\left(4x^k + \frac{a}{x^k}\right)$$
- $$x^{k+1} = \frac{1}{2}\left(x^k + \frac{a}{x^k}\right).$$

Compute their fixed point. Which iteration scheme converges faster to the fixed point?

Exercise 3

In the matlab file called `SD_CG.m` the Steepest Descent (SD) method and the Conjugate Gradient (CG) method are implemented to solve the linear system $Ax = b$. Read the code, answer the questions written in it and describe the main differences between the two methods. Let A a 100×100 symmetric matrix with 1, 2, 3, ... 100 on the diagonal, 1 on the sub- and superdiagonals and zero elsewhere. Let $b = (1, 1, ..., 1)^T$. Run the code using 100 iterations to solve approximately the system $Ax = b$ with both CG and SD respectively. Compare the numerical solutions (obtained with SD and CG) to the reference solution indicated as `xx` in the code. What do you notice? Explain.