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Monte Carlo Methods

Worksheet 9: More Metropolis Algorithms

Exercise 19 (Metropolis–Hastings sampler)**.** Let $\Pi : X \to [0,1]$ be a positive random field and G a stochastic matrix on X. Consider the Metropolis–Hastings sampler

$$\pi(x,y) = \begin{cases} G(x,y)A(x,y), & x \neq y, \\ 1 - \sum_{z \neq x} \pi(x,z), & x = y, \end{cases}$$

with

$$A(x,y) = \min\left\{ \frac{\Pi(y)G(y,x)}{\Pi(x)G(x,y)}, 1 \right\}.$$

Prove that π is reversible with respect to Π.

Exercise 20 (Two configurations)**.** Consider $X = \{1,2\}$, the random field $\Pi = (\frac{2}{3}, \frac{1}{3})$, and the proposal matrix

$$G = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix}.$$

a) Simulate and plot Metropolis trajectories of length $n = 100$ and $n = 10^5$. Plot the corresponding histograms.
b) Simulate and plot trajectories of length $n = 100$ and $n = 10^5$ for the Metropolis–Hastings sampler with acceptance probabilty

$$A(x,y) = \frac{\Pi(y)}{\Pi(x) + \Pi(y)}, \qquad x, y \in X.$$

Plot the corresponding histograms.
c) Formulate the transition matrix π for the methods used in (1) and (2). Calculate the contraction coefficient $c(\pi)$ in both cases.