

# MONTE CARLO METHODS

## Worksheet 8: Gibbs sampler

**Exercise 18** (Gibbs sampler). Consider the two-dimensional Gaussian distribution

$$\Pi(x) = Z^{-1} \exp(-H(x)), \quad x \in \mathbb{R}^2,$$

with  $H(x) = (x_1 - \frac{1}{2}x_2)^2 + (x_2 - \frac{1}{5}x_1)^2$ .

a) Explain why the conditional probability satisfies

$$\Pi(dx_1 | x_2) = \frac{\exp(-H(x_1, x_2))}{\int_{\mathbb{R}} \exp(-H(z_1, x_2)) dz_1} dx_1, \quad x_2 \in \mathbb{R}.$$

b) Run the Gibbs sampler for  $\Pi$  with  $n = 200$  sweeps. Plot the trajectory in the plane as well as its  $x_1$ - and  $x_2$ -components as functions of  $n$ .