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Monte Carlo Methods

Worksheet 5: Stratified and importance sampling

Exercise 13 (Stratified Sampling)**.** Reconsider the integral

$$\int_0^1 x^{\alpha-1} \exp(-x)\, dx, \qquad \alpha \in \{0.51, 0.8\}.$$

Approximate the two integral values by plain direct simulation and stratified sampling over an equidistant partition of $[0, 1]$.

Exercise 14 (Importance sampling)**.** Consider the function

$$f_d(x) = (\mathrm{e} - 1)^{-d} \exp(x_1 + \cdots + x_d), \qquad x \in \Omega = [0, 1]^d,$$

and the integral

$$a(f) = \int_\Omega f(x) dx$$

for integrable $f : \Omega \to \mathbb{R}$.

a) Calculate $a(f_d)$ by hand.

b) Calculate the mean square error for the direct simulation of $a(f_d)$ by uniformly distributed random variables and confirm your result by numerical simulation in dimension $d \in \{1, 5, 10, 20\}$.

c) Construct a random variable $I_n(f)$ based on direct simulation with importance sampling such that

$$\sup \left\{ \sqrt{\mathbb{E}((I_n(f) - a(f))^2)} \mid f : \Omega \to \mathbb{R} \text{ integrable}, |f| \leq f_d \right\} \leq n^{-1/2}.$$

Confirm your result by numerical experiments.