Prof. Dr. Caroline Lasser
Johannes Keller

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Monte Carlo Methods

Worksheet 4: Antithetic sampling and control variates

Exercise 11 (Control variates)**.** Reconsider the integral

$$\int_0^1 f(x)dx = 2 \quad \text{for} \quad f(x) = \exp(\sqrt{x}).$$

a) Approximate the integral by direct simulation, using the function $\widetilde{f}(x) = \exp(x)$ and $b = 1$ for building a control variate.
b) Estimate the optimal parameter b^* using empirical covariances.
c) Repeat the simulation using the estimated optimal parameter and compare your results with the previous ones.

Exercise 12 (Antithetic sampling and control variates)**.** Consider the integral

$$\int_0^1 x^{\alpha-1} \exp(-x)dx, \qquad \alpha = 0.51.$$

Approximate the integral value by the following methods:

a) plain direct simulation,
b) direct simulation with antithetic sampling,
c) direct simulation with a control variate using $\widetilde{f}(x) = x^{\alpha-1}$ and $b = 1$.