

MONTE CARLO METHODS

Worksheet 3: uniform pseudorandom samples

Exercise 9 (Empirical distribution). Let $(X_n)_{n \in \mathbb{N}}$ be i.i.d. with cumulative distribution function $F : \mathbb{R} \rightarrow [0, 1]$, and

$$F_n(x) = \frac{1}{n} |\{j = 1, \dots, n : X_j \leq x\}|, \quad x \in \mathbb{R},$$

the empirical distribution function. Let $x \in \mathbb{R}$ and prove $\lim_{n \rightarrow \infty} F_n(x) = F(x)$ almost surely.

Exercise 10 (Testing). Test the MATLAB built-in `randi` for generating samples uniformly distributed on $\{1, \dots, 4\}$ by performing a chi-square test for $n = 100$ independently generated samples.