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Monte Carlo Methods

Worksheet 2: inversion and rejection method

Exercise 5 (The Laplace density). Consider the Laplace probability density

$$f(x) = \frac{1}{2\sigma} \exp(-|x-\mu|/\sigma)$$

with parameters $\mu \in \mathbb{R}$ and $\sigma > 0$.

a) Compute the cumulative distribution function of f and its inverse.
b) Simulate $n = 1000$ independent Laplace distributed random variables and plot a histogram.
c) Consider the Matlab code

```
n = 1e+3; u = rand(n,1); v = rand(n,1); y = -log(u);
y = sign(v-0.5).*y;
```

Prove that y simulates a Laplace distributed random variable.

Exercise 6 (Normal generator by rejection from the Laplace density). Employ the inequality

$$\frac{1}{\sqrt{2\pi}} e^{-x^2/2} \leq \sqrt{\frac{e}{2\pi}}\, e^{-|x|} \quad \forall x \in \mathbb{R},$$

and apply the rejection method introduced in the lecture in order to sample from the standard normal distribution.

a) Simulate $n = 1000$ independent normally distributed random variables by rejection from the Laplace distribution and plot a histogram.
b) Simulate the expected number of iterations by direct simulation.

Exercise 7 (Normal generator by rejection from the exponential distribution). Consider the following rejection method: Generate an exponentially distributed random variable X and an independent variable V uniformly distributed on $[-1, 1]$. If $(1 - X)^2 > -2\log(|V|)$, then repeat the same, otherwise return $\text{sign}(V)X$.

a) Prove that the algorithm's output is a normally distributed random variable.
b) Simulate $n = 1000$ independent samples by this method and deduce the expected number of iterations.

Exercise 8 (Another rejection method). Consider the following method: Generate a random variable X with density g and an independent variable Y with continuous cumulative distribution function Ψ. If $X > Y$, then repeat the same, otherwise return X.

Prove that the algorithm's output is a random variable with density $cg(1 - \Psi)$, where $c > 0$ is a positive constant.