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Monte Carlo Methods

Worksheet 11: Convergence test and quasi-Monte Carlo quadrature

Exercise 23 (Rayleigh–Ritz principle)**.** Let $\mathcal{H}$ be a finite-dimensional Hilbert space with inner product $\langle\cdot,\cdot\rangle$. Let $A : \mathcal{H} \to \mathcal{H}$ be a linear selfadjoint operator. Deduce from the spectral theorem that

$$\min \sigma(H) = \min \left\{ \frac{\langle x, Hv\rangle}{\langle v, v\rangle} : v \in \mathcal{H} \setminus \{0\} \right\}.$$

Exercise 24 (Gelman–Rubin test)**.** Assume that each of the m Markov chains satisfies the strong law of large numbers. Prove that $\hat{V}_n$ is a strongly consistent estimator for $\mathbb{V}_\mu(f)$:

$$\lim_{n\to\infty} \hat{V}_n = \mathbb{V}_\mu(f) \quad \text{a.e.}$$

Exercise 25 (Quasi-Monte Carlo quadrature)**.** Consider the standard normal distribution μ on $\mathbb{R}^{10}$ and its mean

$$a = \int_{\mathbb{R}^d} x d\mu(x) = 0.$$

a) Compute a by direct simulation using up to $n = 10^4$ i.i.d. samples.
b) Compute a by quasi-Monte Carlo quadrature using up to $n = 10^4$ Halton points.
c) Compute a by quasi-Monte Carlo quadrature using up to $n = 10^4$ Sobol points.

Plot the error of all three simulations as a function of n and compare with the theoretically expected rate.