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Monte Carlo Methods

Worksheet 10: Convergence Rates

Exercise 21 (Positivity of the stationary distribution)**.** Let P be a finite positive stochastic matrix that is reversible with respect to μ. Prove, that every component of μ is positive.

Exercise 22 (Mihail's identity)**.** Let P be a finite stochastic matrix, that is reversible with respect to μ. Prove

$$\mathbb{V}_\mu(Pv) = \mathbb{V}_\mu(v) - \langle(\mathrm{Id} - P^2)v, v\rangle_\mu$$

a) for all vectors v with $\mathbb{E}_\mu(v) = 0$,
b) for all vectors v.