The SIAM 100-Digit Challenge

A Study in High-Accuracy Numerical Computing

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Errata, Amendments, Improvements — *(Last updated 26 January 2006)*

Page x line 6 ——— (FB) 18 Jul 2004.

Only digits were required, neither proofs of existence and uniqueness of the solution, convergence of the method, nor correctness of the result; nevertheless ⇝ Only digits were required, no proofs: not of the existence and uniqueness of the answer, not of the convergence of the method, not of the correctness of the result. Nevertheless

Page xi line 10 ——— (FB) 30 Nov 2004.

thanks to John Boersma ⇝ thanks to the late John Boersma (1937–2004)

Page 1 epigram ——— (SW) 09 Jul 2004.

break line after the word "attack", second line should begin with a capital, no punctuation at the end

▸**Page 19** line −11 ——— (SW) 18 Jun 2005.

next section ⇝ §1.5

▸**Page 22** line 7 ——— (FB) 16 Aug 2004.

`(1+I*(1-2*t)))` ⇝ `(1+I*(1-2*t))))`

▸**Page 74** line 1 of caption of Figure 3.4 ——— (DL) 06 Sep 2004.

Loci of poles (dashed lines) ⇝ Loci of poles (dot-dashed lines)

▸**Page 74** line 2 of caption of Figure 3.4 ——— (DL) 06 Sep 2004.

The dotted ⇝ The dashed

▸**Page 85** line 14 ——— (LP) 14 Oct 2004.

$\min_{T\in\mathcal{R}}(f[T])$ ⇝ $\min_{T\in\mathcal{R}}(\text{right end of } f[T])$

▸**Page 85** line 19 ——— (LP) 14 Oct 2004.

$\min_{T\in\mathcal{R}}(f[T])$ ⇝ $\min_{T\in\mathcal{R}}(\text{left end of } f[T])$

▸**Page 86** line 8 ——— (FB) 08 Sep 2004.

-3.30687^{49}_{56} ⇝ -3.30686864747^{49}_{56}

Page 92 line −19 ——— (LP) 14 Oct 2004.

has a zero in $[a,b]$. ⇝ has a zero in $[a,b] = X$. By (a) this zero lies in $N(X)$, too.

Page 107 line 12 ——— (DL) 14 Jul 2004.

[Rut90, Thms. 7.4 and 7.5] ⇝ [Rut90, Thms. 7.4 and 7.5])

Page 123 line −5 ——— (FB) 18 Jan 2005.

Now, it is reasonable to expect $\leadsto$ Now, in the transient case it is reasonable to expect

Page 124 line 1 ——— (FB) 18 Jan 2005.

One can prove the convergence $\leadsto$ For transient random walks one can prove the convergence

Page 133 add footnote to line −1 ——— (FB) 12 Jun 2005.

In particular, the success of these calculations shows that using the three-term recurrence as advocated for the question at hand is numerically stable. According to the general theory of computational aspects of three-term recurrences (see [Gau67]), the observed numerical stability is equivalent to saying that (p_k) is a *dominant* solution of the three-term recurrence (6.14). In fact, for $p_{\mathrm{EW}} \cdot p_{\mathrm{NS}} \neq 0$, by a theorem of Poincaré (see [Gau67, p. 34]), we have $\lim_{k\to\infty} p_{k+1}/p_k = \lambda_{1,2}$ with $\lambda_{1,2} = 4(p_{\mathrm{EW}} \pm p_{\mathrm{NS}})^2$ being the roots of the characteristic equation

$$\lambda^2 - 8(p_{\mathrm{EW}}^2 + p_{\mathrm{NS}}^2) + 16(p_{\mathrm{EW}}^2 - p_{\mathrm{NS}}^2)^2 = 0.$$

Now, (p_k) is dominant if and only if the limit is the *larger* root $\lambda_1 = \sigma^2$. We will prove this to be the case in part (b) of the subsequent Lemma 6.1.

▸**Page 145** last displayed equation ——— (BL) 13 Jan 2005.

$|R_n| \leq 2e^{1/4x} \frac{(2n-1)!!^2}{n!(8x)^n}$ $\leadsto$ $|R_n| \leq 2e^{\pi/8x} \frac{\Gamma^2(n+1/2)}{\Gamma^2((n+1)/2)(4x)^n}$

▸**Page 157** line −13 ——— (FB) 09 Jun 2005.

iterations that are necessary $\leadsto$ iterations that are sufficient

Page 164 line 19 ——— (FB) 30 Nov 2005.

add the following footnote after the word "approaches": Zhendong Wan has found a completely new approach that calculates the rational solution in a stunning 10 minutes of computing time. For details see his paper *Exactly Solve Integer Linear Systems Using Numerical Methods*, forthcoming in J. Symb. Comput., and the web page for this book.

▸**Page 177** label of the vertical axis of Figure 8.2 (left part) ——— (FB) 14 Jul 2004.

(q) $\leadsto$ $\theta(q)$

▸**Page 177** label of the vertical axis of Figure 8.2 (right part) ——— (FB) 14 Jul 2004.

$(e^{-\pi^2 t})$ $\leadsto$ $\theta(e^{-\pi^2 t})$

▸**Page 182** lines −13 and −1 ——— (AS) 25 Jan 2005.

Figure 9.3 $\leadsto$ Figure 9.2

▸**Page 182** last displayed formula ——— (FB) 26 Jan 2006.

$f_0(z(t,\alpha))$ $\leadsto$ $f_0(z(t),\alpha)$

▸**Page 183** line 2 ——— (AS) 25 Jan 2005.

$f_1(t,\pi/4)$ $\leadsto$ $f_1(x,\pi/4)$

Page 184 2nd display, 1st line — (AS) 25 Jan 2005.

$d_n = \frac{f(x_{n+1})-f(x_n)}{h_n}, \quad \theta_n = \frac{d_n}{2(d_{n-1}-d_n)} \quad \leadsto$

$d_{n-1} = \frac{f(x_n)-f(x_{n-1})}{h_{n-1}}, \quad \theta_{n-1} = \frac{d_{n-1}}{2(d_{n-2}-d_{n-1})}$

Page 194 add sentence in line −3 — (FB) 08 Feb 2005.

The algorithm that leads *Mathematica* and Maple to this surprise is explained at length in [Mar83].

Page 200 line −2 — (FB) 09 Jul 2004.

we look at the 10×1 rectangle, the 1×1 rectangle, and the $\sqrt{3} \times 1$ rectangle $\leadsto$ we look at, in addition to the parameters given in the contest, the 1×1 and $\sqrt{3} \times 1$ rectangles

▸**Page 221** line 1 — (FB) 08 Jul 2004.

[Har40, p. 228] $\leadsto$ [Har40, p. 229]

Page 223 add sentence to footnote 83 — (FB) 18 Jul 2004.

Interestingly, Driscoll himself did not actually use his own software to solve Problem 10; see his remark quoted on p. 13.

▸**Page 231** line −6 — (DL) 14 Jul 2004.

matrix { = $[\phi_{k,j}]$ $\leadsto$ matrix $\Phi = [\phi_{k,j}]$

▸**Page 232** line 4 — (DL) 14 Jul 2004.

the matrix { $\leadsto$ the matrix Φ

▸**Page 232** line 18 — (DL) 14 Jul 2004.

$\phi k,\, j = \alpha k\, \beta k,\, j$, where αk is allowed $\leadsto$ $\phi_{k,j} = \alpha_k\, \beta_{k,j}$, where α_k is allowed

▸**Page 233** line −14 — (DL) 14 Jul 2004.

modified matrix { $\leadsto$ modified matrix Φ

▸**Page 234** line 3 — (FB) 06 Nov 2004.

$$s_{k,n} = \sum_{i=1}^{k-1} a_i + \frac{1}{1-r} \sum_{j=0}^{n} \left(\frac{r}{1-r}\right)^j \Delta^j b_k \qquad \leadsto$$

$$s_{k,n} = \sum_{i=1}^{k} a_i + \frac{r^k}{1-r} \sum_{j=0}^{n-1} \left(\frac{r}{1-r}\right)^j \Delta^j b_{k+1}$$

▸**Page 234** line 11 — (FB) 06 Nov 2004.

$c_{n-1}k^{n-1} \leadsto c_{m-1}k^{m-1}$

▸**Page 234** line 13 — (FB) 06 Nov 2004.

$c_n k^n \leadsto c_{m-1}k^{m-1}$

▸**Page 234** lines −10 and −11 — (FB) 06 Nov 2004.

$\Delta^k c_1 \leadsto \Delta^k b_1$

Page 257 line −17 ——— (DL) 14 Jul 2004.

Rodriguez, Villegas, and Zagier ⇝ Rodriguez Villegas, and Zagier

▶**Page 257** line −14 ——— (BL) 07 Nov 2004.

totally accelerating sequences ⇝ totally oscillating sequences

▶**Page 284** 2nd display, 2nd equation ——— (SW) 09 Jan 2005.

$0.7y - 0.1z$ ⇝ $0.7x - 0.1z$

▶**Page 285** line 3 ——— (ES) 03 Jan 2005.

$-\mathrm{div}(c(x)\,\mathrm{grad}\,u(x)) = 1$ ⇝ $-\mathrm{div}(a(x)\,\mathrm{grad}\,u(x)) = 1$

Page 291 add entry after line 4 ——— (FB) 12 Jun 2005.

[Gau67] Walter Gautschi, *Computational aspects of three-term recurrence relations*, SIAM Review **9** (1967), 24–82. (Cited on p. 133.)

Page 294 add entry after line 6 ——— (FB) 08 Feb 2005.

[Mar83] Oleg I. Marichev, *Handbook of Integral Transforms of Higher Transcendental Functions: Theory and Algorithmic Tables*, Ellis Horwood, Chichester, 1983. (Cited on p. 194.)

Page 294 entry [Mon56] ——— (JB) 14 Sep 2004.

Cited on p. 145 ⇝ Cited on pp. 144, 145.

Page 295 entry [Olv74] ——— (JB) 14 Sep 2004.

Cited on pp. 70, 277 ⇝ Cited on pp. 70, 144, 145, 227.