42) $$kk\left(\frac{2K}{\pi}\right)^3 = 16\left\{\frac{q}{1+q^2} + \frac{4q^2}{1+q^4} + \frac{9q^3}{1+q^6} + \frac{16q^4}{1+q^8} + \ldots\right\} =$$
$$16\left\{\frac{q(1+q)}{(1-q)^3} - \frac{q^3(1+q^3)}{(1-q^3)^3} + \frac{q^5(1+q^5)}{(1-q^5)^3} + \ldots\right\}.$$

Ex his posito $-q$ loco q obtinemus:

43) $$kk'k'\left(\frac{2K}{\pi}\right)^3 = 4\left\{\frac{\sqrt{q}}{1-q} - \frac{9\sqrt{q^3}}{1-q^3} + \frac{25\sqrt{q^5}}{1-q^5} - \frac{49\sqrt{q^7}}{1-q^7} + \ldots\right\}$$

44) $$k'k'\left(\frac{2K}{\pi}\right)^3 = 1 - 4\left\{\frac{q}{1-q} - \frac{9q^3}{1-q^3} + \frac{25q^5}{1-q^5} - \frac{49q^7}{1-q^7} + \ldots\right\}$$

45) $$k'kk\left(\frac{2K}{\pi}\right)^3 = 16\left\{\frac{q}{1+q^2} - \frac{4q^2}{1+q^4} + \frac{9q^3}{1+q^6} - \frac{16q^4}{1+q^8} + \ldots\right\}.$$

Formulis 42), 44) additis, obtinemus $\left(\frac{2K}{\pi}\right)^3$; 40) et 43), 41) et 45) subductis obtinemus $\left(\frac{2kK}{\pi}\right)^3$, $\left(\frac{2k'K}{\pi}\right)^3$, e quibus posito resp. $\sqrt{q}$, q^2 loco q prodit $\left(\frac{4\sqrt{k}K}{\pi}\right)^3$, $\left(\frac{2\sqrt{k'}K}{\pi}\right)^3$; e $\left(\frac{4\sqrt{k}K}{\pi}\right)^3$ posito $-q$ loco q obtinetur $\left(\frac{4\sqrt{kk'}K}{\pi}\right)^3$.

Sub finem, posito $k = \sin\vartheta$, evolvamus ipsum $\vartheta = \text{Arc.}\sin k$. Vidimus, posito $\sqrt{q}$ loco q abire k' in $\frac{1-k}{1+k}$; ponamus rursus $-q$ loco q, abit k in $\frac{ik}{k'}$, sive in $i.\tang\vartheta$; ita ut posito $i\sqrt{q}$ loco q, expressio $\frac{-\log k'}{2i}$ mutetur in

$$-\frac{1}{2i}\log\left(\frac{1-i\tang\vartheta}{1+i\tang\vartheta}\right) = \vartheta.$$

Hinc e formula 2)

$$-\log k' = \frac{8q}{1-q^2} + \frac{8q^3}{3(1-q^6)} + \frac{8q^5}{5(1-q^{10})} + \frac{8q^7}{7(1-q^{14})} + \ldots$$

eruimus:

46) $$\vartheta = \text{Arc}\sin k = \frac{4\sqrt{q}}{1+q} - \frac{4\sqrt{q^3}}{3(1+q^3)} + \frac{4\sqrt{q^5}}{5(1+q^5)} - \frac{4\sqrt{q^7}}{7(1+q^7)} + \ldots,$$

quae in hanc facile transformatur:

47) $$\frac{\vartheta}{4} = \text{Arc tg}\sqrt{q} - \text{Arc tg}\sqrt{q^3} + \text{Arc tg}\sqrt{q^5} - \text{Arc tg}\sqrt{q^7} + \ldots,$$

quae inter formulas elegantissimas censeri debet.