

A Note on Two Problems in Connexion with Graphs

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We consider n points (nodes), some or all pairs of which are connected by a branch; the length of each branch is given. We restrict ourselves to the case where at least one path exists between any two nodes. We now consider two problems.

Problem 1. Construct the tree of minimum total length between the n nodes. (A tree is a graph with one and only one path between every two nodes.)

In the course of the construction that we present here, the branches are subdivided into three sets:

I. the branches definitely assigned to the tree under construction (they will form a subtree);

II. the branches from which the next branch to be added to set I, will be selected;

III. the remaining branches (rejected or not yet considered).

The nodes are subdivided into two sets:

A. the nodes connected by the branches of set I,

B. the remaining nodes (one and only one branch of set II will lead to each of these nodes).

We start the construction by choosing an arbitrary node as the only member of set A, and by placing all branches that end in this node in set II. To start with, set I is empty. From then onwards we perform the following two steps repeatedly.

Step 1. The shortest branch of set II is removed from this set and added to set I. As a result one node is transferred from set B to set A.

Step 2. Consider the branches leading from the node, that has just been transferred to set A, to the nodes that are still in set B. If the branch under consideration is longer than the corresponding branch in set II, it is rejected; if it is shorter, it replaces the corresponding branch in set II, and the latter is rejected.

We then return to step 1 and repeat the process until sets II and B are empty. The branches in set I form the tree required.

The solution given here is to be preferred to the solution given by J. B. KRUSKAL [1] and those given by H. LOBERMAN and A. WEINBERGER [2]. In their solutions all the — possibly $\frac{1}{2}n(n-1)$ — branches are first of all sorted according to length. Even if the length of the branches is a computable function of the node coordinates, their methods demand that data for all branches are stored simultaneously. Our method only requires the simultaneous storing of