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Exercises 1 and 2 are to be handed in on Thursday, 03.02.2011, before the lecture.

Exercise 1 (Least Squares Method)
The following table shows the values of the deformation ϵ of a biological tissue to which the stress σ is applied:

ϵ	0	0.06	0.14	0.25	0.31	0.47	0.60	0.70
σ	0	0.08	0.14	0.20	0.23	0.25	0.28	0.29

Use the least squares method to find the regression line for this data, i.e. a linear polynimial $p(\epsilon) = a_1\epsilon + a_0$ that minimizes $\sum_{j=1}^{8} |p(\epsilon_j) - \sigma_j|^2$.

(a) Formulate the overdetermined linear system in the form $Ax = b$.

(b) Solve the problem via normal equations and via singular value decomposition.

(c) Compare your results.

Exercise 2 (Gauss-Seidel Iteration) ($*$)
Following the steps for the forward Gauss-Seidel iteration presented in the lecture derive the so-called backward Gauss-Seidel iteration by choosing $A_1 = D + U$, $A_2 = L$ (D diagonal, L lower triangular with zeros on the diagonal, U upper triangular with zeros on the diagonal).

Exercise 3 (LDU Factorization) ($*$)
Let us consider a linear system $Ax = b$, where b is chosen in such a way that the solution is the unit vector $(1, 1, ..., 1)^T$ and A is the 10×10 tridiagonal matrix whose diagonal entries are all equal to 3, the entries of the first lower diagonal are equal to -2 and those of the upper diagonal are all equal to -1. Matlab code:

```
n=10;
A=3* eye(n)-2* diag(ones(n-1,1),-1)- diag(ones(n-1,1),1);
b=A*ones(n ,1);
```

Compare the number of iteration steps required for Richardson, Jacobi, forward and backward Gauss-Seidel iterations to get the approximation error less than $tol = 10^{-12}$.