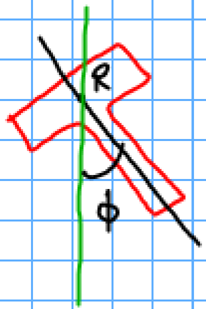
(1.8) Robot navigation



$\Omega \times [0,2\pi]$ state space $\Omega \subset \mathbb{R}^2$

$$T(x,\Theta) = \min_{\gamma} \int_{(x_0,\theta_0)}^{(x,\theta)} v^{-1}\, ds$$

how to evaluate ?

$$\boxed{v(x,\Theta) = \begin{cases} 1 & \text{Robot @ } (x,\Theta) \text{ does not intersect obstacles} \\ 0 & \text{otherwise} \end{cases}}$$

eikonal equation

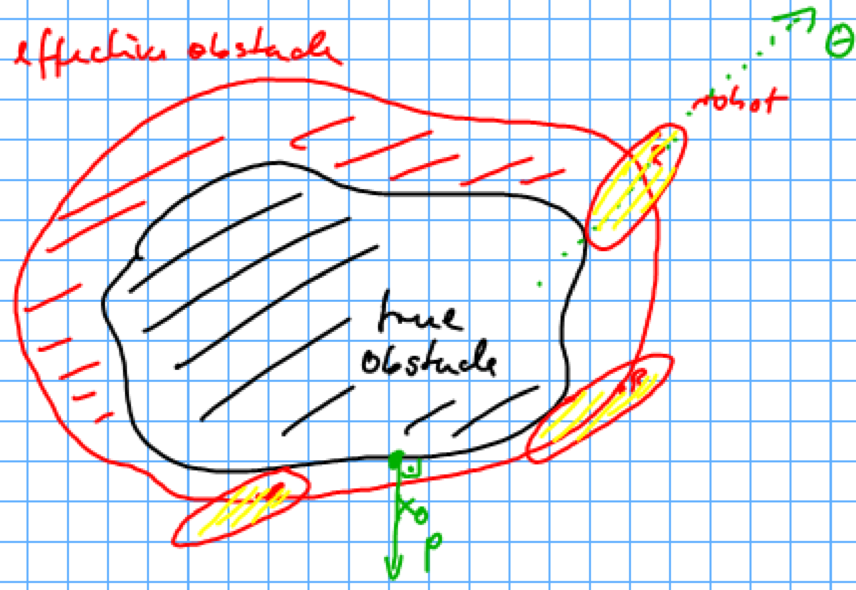
$$v \cdot |\nabla T| = 1 \qquad T(x_0,\theta_0) = 0$$

$$v(x,\Theta)\sqrt{\left(\frac{\partial T}{\partial x_1}\right)^2 + \left(\frac{\partial T}{\partial x_2}\right)^2 + \left(\frac{\partial T}{\partial \Theta}\right)^2} = 1$$

(1.9) Effective obstacles

fixed ①

robot $\cap$ obstacles $= \emptyset \iff R \notin$ effective obstacles



given boundary of "true" obstacle

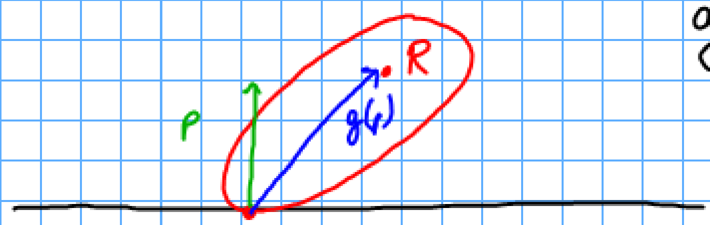
$$\{ H(x) = 0 \}$$

effective boundary

$$\{ H(x) = 1 \}$$

Convex shaped robot

$$g(tp) = g(p) \qquad \forall t > 0$$



ray $\qquad x_t = x_0 + t\, g(p)$ $\qquad$ p normal of true boundary @ x_0

defining relation $\qquad H(x_0 + t\, g(p)) = t$

differentiate $\qquad \nabla H(x_0 + t\, g(p)) \cdot g(p) = 1$ $\qquad p = \nabla H(x_0)$

$$= \nabla H(x_0 + t\, g(\nabla H(x_0))\; g(\nabla H(x_0))$$

$$= \nabla H(x)\; g(\nabla H(x))$$

moving along ray is translation, does not change g

$$\underbrace{1 = \nabla H(x) \cdot g(\nabla H(x))}_{\text{PDE}}$$

$$\underbrace{H\big|_{\text{boundary of obstacle}} = 0}_{\text{boundary condition}}$$

$$= |\nabla H(x)| \underbrace{\frac{\nabla H(x) \cdot g(\nabla H(x))}{|\nabla H(x)|}}_{v(\nabla H(x))}$$

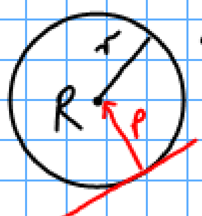
direction-dependent velocity

$$1 = |\nabla H(x)| \cdot v$$

eikonal equation

$$\boxed{\begin{array}{ll} H(x) \le 1 & x \in \text{effective obstacle} \\ H(x) > 1 & x \notin \text{effective obstacle} \end{array}}$$

Example:



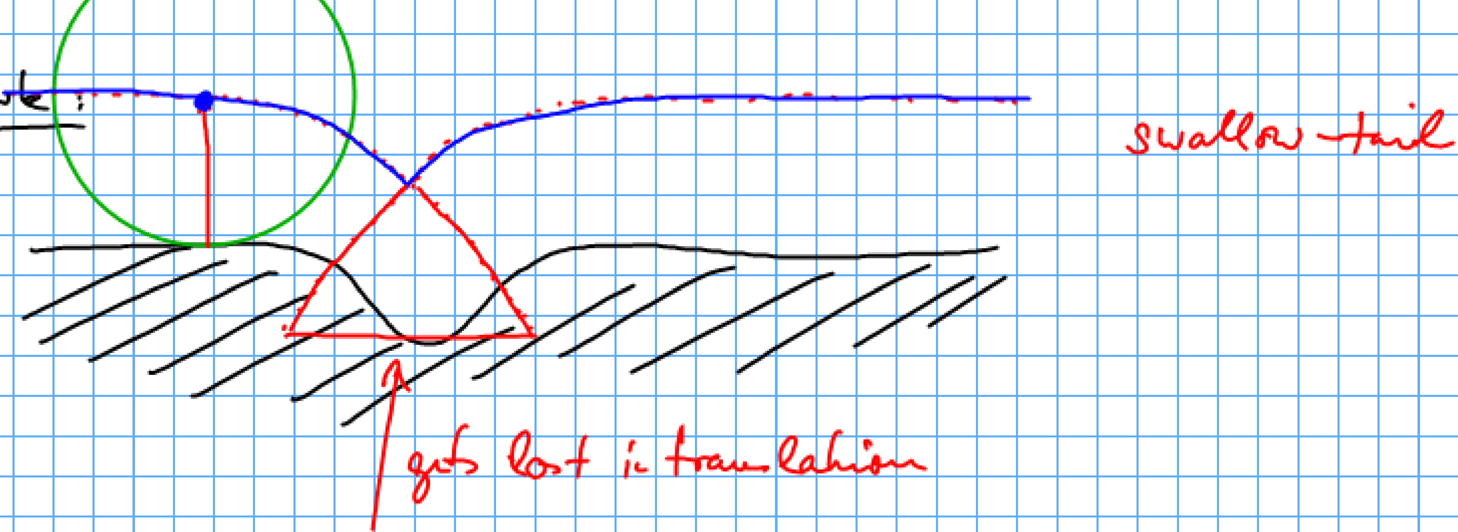
robot

$$g(p) = r \frac{p}{|p|}$$

eikonal equation for effective boundary: $$1 = |\nabla H(x)| \underbrace{\frac{\nabla H(x) \cdot r \cdot \frac{\nabla H(x)}{|\nabla H(x)|}}{|\nabla H(x)|}}_{= r}$$

$$1 = |\nabla H(x)| \cdot r$$

Remark:

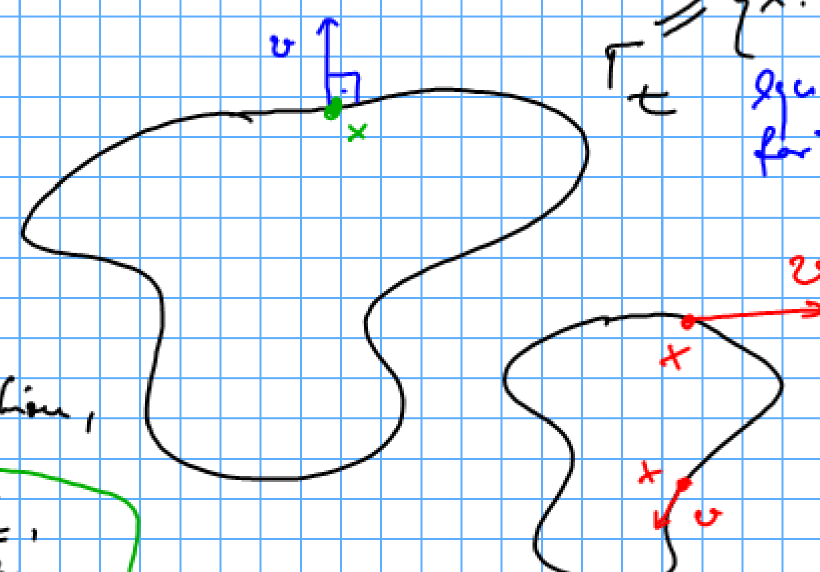


(1.10) Moving phase boundaries

Part: normal velocity $v \geq 0$

$\Gamma_t = \{x : T(x) = t\}$

equation for T ?



$v = v($ local information, global information,

x, direction of normal, curvature of Γ_t

length of Γ_t, shape, topology

parameters, $t)$

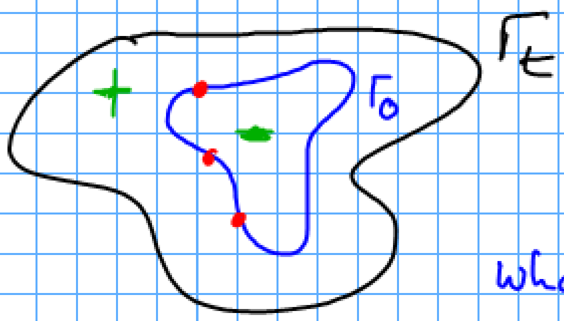
tangential components would just map Γ_t to itself, does not change geometry.

$1 = |\nabla T| \cdot v$ eikonal equation $\qquad T|_{\Gamma_0} = 0$

<u>Question</u>: what to do for general v (both signs possible)?

$\varphi(x,t) = T(x) - t$

§2 Level-Set-Equation



$\Gamma_t = \{ x : \varphi(x,t) = 0 \}$

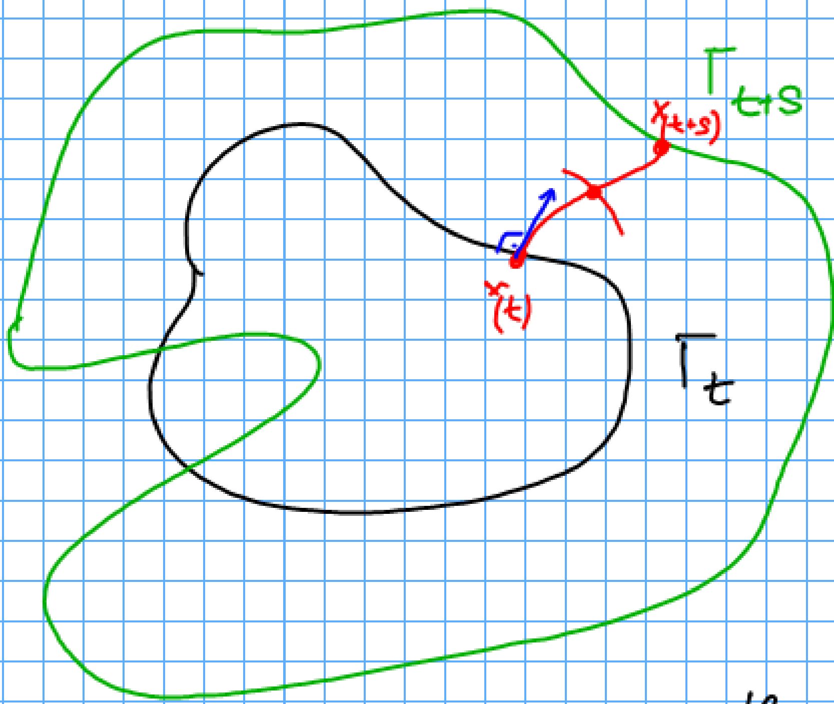
↑ equation for φ ?

what is $\varphi(x,0)$?

$f(x) = \varphi(x,0) = \text{signed distance}(x, \Gamma_0)$

$$\boxed{|\nabla f| = 1 \qquad f|_{\Gamma_0} = 0}$$

eikonal



$$\varphi(x(t), t) = 0$$

$$\varphi_t + \nabla\varphi \cdot \dot{x} = 0$$

$$|\nabla\varphi| \, (\dot{x}(t) \cdot n) = (v) \cdot |\nabla\varphi|$$

outward normal (unit normal)

$$n \parallel \nabla\varphi$$

therefore $\quad \nabla\varphi \cdot \dot{x} = |\nabla\varphi| \cdot v$

$$\nabla = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_d}\right)$$

$$\boxed{\varphi_t + v \cdot |\nabla\varphi| = 0}$$

$$\varphi\big|_{t=0} = \varphi_0$$

level set equation (Osher, Sethian 1988)

Connection to eikonal equation for $v \geq 0$

$$\boxed{v\,|\nabla T| = 1}$$

$$\Gamma_t = \{ x : T(x) = t \}$$
$$= \{ x : \varphi(x,t) = 0 \}$$

$$\varphi(x,t) = T(x) - t$$

$$\varphi_t = -1$$

$$\nabla\varphi = \nabla T$$

$$\varphi_t + v \cdot |\nabla\varphi| = 0$$

$$-1 + v\,|\nabla T| = 0$$