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On a uniparametric family of strongly A-stable Runge-Kutta collocation methods for stiff systems and DAEs.

Domingo Hernández-Abreu.
Universidad de La Laguna (Spain).
dhabreu@ull.es

Abstract

In this talk we shall describe the main features of a newly defined one-parameter family of high order s-stage Runge-Kutta methods, which have been recently introduced in [1,2] for the numerical integration of ordinary differential equations [4]. This family of methods includes the well-known family of $LobattoIIIA$ methods, and it is based on the characterization of $s-$point interpolatory quadrature rules of Precision Degree $q = 2s - 4$, having two prefixed nodes $c_1 = 0$ and $c_s = 1$. For each $s \geq 3$, the so-called $SAFERK(\alpha)$ methods are A-stable collocation Runge-Kutta methods possessing a first internal stage of explicit type and coefficients depending on a free parameter α. Moreover, they have the same number of implicit stages as the $s-$stage $LobattoIIIA$ and $(s-1)-$stage $RadauIIA$ methods.

First, regarding non-stiff problems, it is shown that those $SAFERK$ methods with positive weights and nodes located in the integration interval possess, in the l_2-norm, a smaller principal term of local error than the $(s-1)-$stage $RadauIIA$ method. Next, we derive global error estimates on finite intervals for $SAFERK(\alpha)$ methods when applied to several kinds of stiff semilinear problems, differential-algebraic equations (DAEs) [1,2] and singularly perturbed systems [3]. In particular, it is shown that strongly A-stable $SAFERK(\alpha, s)$ methods possess a higher stiff order than the $RadauIIA(s-1)$ method, since the new methods possess a higher stage order.

Numerical illustrations involving the $RadauIIA(3)$, $LobattoIIIA(4)$ and some specific $SAFERK(\alpha, 4)$ method will be presented.

References

[1] S. González-Pinto, D. Hernández-Abreu, J.I. Montijano, *An efficient family of strongly A-stable Runge-Kutta collocation methods for stiff systems and DAE's. Part I: stability and order results.* J. Comput. Appl. Math. 234 (2010), 1105-1116.

[2] S. González-Pinto, D. Hernández-Abreu, J.I. Montijano, *An efficient family of strongly A-stable Runge-Kutta collocation methods for stiff systems and DAE's. Part II: convergence results.* To appear in Appl. Numer. Math.

[3] S. González-Pinto, D. Hernández-Abreu, *Global error estimates for a uniparametric family of stiffly accurate Runge-Kutta collocation methods on singularly perturbed problems.* In preparation.

[4] E. Hairer, G. Wanner, *Solving Ordinary Differential Equations II. Stiff and differential-algebraic problems*, Springer–Verlag, second edition (1996).