MOCK EXAM FOR NUMERICAL PROGRAMMING II

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1. Numerical tools: basics

Please provide a short informative description for each topic (1–2 sentences).

(1) 5-point stencil
(2) Hat (or tent) functions
(3) Linear multistep method
(4) Hyperbolic cross

2. Numerical programs: reading

Please answer each question in 1–2 sentences.

(1) Which mathematical problem is solved by the program below?
(2) Which numerical method is used?
(3) How accurate is the program's solution?

```
x = linspace(0,1,n+2);
h = x(2)-x(1);
e = ones(n,1);
A = -spdiags([e,-2*e,e],-1:1,n,n)/h^2;
u = A\b;
plot(x,[0;u;0],'b-')
```

3. Numerical programs: writing

Please provide a program outline for one time-step of a Strang splitting with Fourier collocation for solving the one-dimensional time-dependent Schrödinger equation

$$i\partial_t \psi = (T + V)\psi.$$

(We assume that $\psi(t, \cdot)$ *is negligible outside the interval* $[-\pi, \pi]$*.)*

4. Numerical methods: evaluating

Please say yes or no, and provide one explaining sentence for your judgement.

(1) FEM discretizations of elliptic equations produce sparse linear systems.
(2) If the fine grid matrix is symmetric, then full reweighting results in a symmetric coarse grid representation.
(3) Discretizing the heat equation by centered finite differences and the implicit Euler method imposes step-size restrictions.
(4) The Chebyshev method is a time-reversible method for solving the time-dependent Schrödinger equation.

5. Numerical methods: deriving

The one-dimensional diffusion equation

$$-(\sigma(x)u(x)')' = f(x) \quad \text{on} \quad]0,\pi[$$

with homogeneous Dirichlet boundary conditions, uniformly positive diffusion coefficient $\sigma(x) > 0$ and right hand side f is discretized by a Galerkin method. The resulting linear system reads

$$A_h u_h = b_h.$$

$\triangleright$ Please provide short descriptions for A_h, u_h, and b_h (one to two sentences for each of the three objects).