Numerical Programming 1 (CSE) 2014

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Solutions for Worksheet 9, Exercises 1 and 2

Exercise 1

- In this case,
$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^2}{2x_k} = \frac{x_k}{2},$$
so $x_k = \frac{x_0}{2^k}$, i.e. linear convergence

- In this case,
$$\phi(x_k) = x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^3 - 2x_k + 2}{3x_k^2 - 2}$$
and one has $\phi(0) = 1$ and $\phi(1) = 0$, so for the initial value 0 get the alternating sequence $0, 1, 0, 1, \ldots$, so no convergence.

Exercise 2

Let $\phi_1(x) = \frac{1}{5}\left(4x + \frac{a}{x}\right)$ and $\phi_2(x) = \frac{1}{2}\left(x + \frac{a}{x}\right)$ denote the iteration functions. By setting $x_k = x_{k+1} = x$ and solving $\phi_{1,2}(x) = x$ one sees that the fixed point in both cases is $\sqrt{a}$. The convergence of the algorithm is characterized by $\phi_1'(\sqrt{a}) = \frac{3}{5}$ and $\phi_2'(\sqrt{a}) = 0$. A value between -1 and 1 is sufficient for local convergence (i.e. convergence for initial values close to the fixed point). So we have local convergence in both cases. Since $\phi_2'(\sqrt{a}) = 0$, the convergence of the iteration with ϕ_2 is quadratic, as one sees from the Taylor series: $\phi_2(\sqrt{a} + h) = \sqrt{a} + \frac{1}{2}\phi_2''(\sqrt{a})h^2 + o(h^2) \approx \sqrt{a} + const\ h^2$.