Numerical Programming 1 (CSE) 2014

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Worksheet 8

Exercise 1

A planet follows an elliptical orbit that can be described by:

$$ay^2 + bxy + cx + dy + e = x^2, a, b, c, d, e \in \mathbb{R}.$$

- Find the quadratic form that best approximates the movement of a planet in the sense of least squares (via normal equations) when the following observations are given:

  ```
  x=[1.02,0.95,0.87,0.77,0.67,0.56,0.44,0.30,0.16,0.01];
  y=[0.39,0.32,0.27,0.22,0.18,0.15,0.13,0.12,0.13,0.15];
  ```

  Plot the given data points and the resulting orbit (for example by using a contour-plot).

- Why is this problem nearly rank deficient? To see the effect of this in the least squares approximation perturb the coordinates of each data point by a random number uniformly distributed on [-0.005,0.005]. Plot the new orbit and describe what you observe.

Exercise 2

Following the steps for the forward Gauss-Seidel iteration presented in the lecture derive the so-called backward Gauss-Seidel iteration by choosing $M = D + U, N = -L$ (D diagonal, L lower triangular with zeros on the diagonal, U upper triangular with zeros on the diagonal).

Exercise 3

Let us consider a linear system $Ax = b$, where b is chosen in such a way that the solution is the unit vector $(1, 1, ..., 1)^T$ and A is the 100 x 100 tridiagonal matrix whose diagonal entries are all equal to 3, the entries of the first lower diagonal are equal to -2 and those of the upper diagonal are all equal to -1. Matlab code:

```
n = 100;
A = 4*eye(n) - 2*diag(ones(n-1,1),-1) - diag(ones(n-1,1),1);
b = A*ones(n,1);
```

Compare the number of iteration steps required for Richardson, Jacobi, forward and backward Gauss-Seidel iterations to get the approximation error less than $tol = 10^{-12}$. Plot the error of each method over the number of steps (choose a reasonable scaling of the axes!). Compare the convergence rate with $\|M^{-1}N\|$. (The Richardson iteration results from the splitting $A = M - N, M = I_n, N = I_n - A$.)