Numerical Programming 1 (CSE) 2014

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Worksheet 7

Exercise 1

- Show that, for any matrix norm, holds $\| A \| \geq |\lambda_{max}|$, where $|\lambda_{max}|$ is the biggest eigenvalue of the square matrix A.
- Given A and B non singular square matrices, show that $k(AB) \leq k(A)k(B)$ where $k(A)$ is the condition number of the matrix A.

Exercise 2

Let V be the vector space of the polynomials of degree less or equal than two. Given $p, q \in V$ let's define the inner product on V as

$$\langle p, q \rangle = \frac{1}{2} \int_{-1}^{1} p(x)q(x)dx. \tag{1}$$

By considering the base $\{1, x, x^2\}$ on V, use the Gram-Schmidt procedure to find an orthonormal base $\{v_1, v_2, v_3\}$ of V. What is the vector representation of $1, x$ and x^2 in the base $\{v_1, v_2, v_3\}$?. Find the QR decomposition of the 3×3 matrix whose columns are the 3 vectors representing the polynomials $1, x, x^2$.

Find the second degree polynomial solving the following minimization problem

$$\min_{p \in V} \int_{-1}^{1} \left(p(x) - x^3\right)^2 dx$$

Exercise 3

Given the column vector $v \in \mathbb{R}^n$, compute the eigenvalues and the determinant of the Housholder reflector

$$H_v = I - 2\frac{vv^T}{v^T v}.$$