Numerical Programming 1 (CSE) 2014

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Worksheet 5

Exercise 1

Determine the number of nodes required to approximate

$$\int_0^2 \frac{1}{x+4}dx$$

within an error of 10^{-6} for each error formulae relative to:

- The composite midpoint rule
- The composite trapezoidal rule
- The composite Simpson rule

Exercise 2

Given $y_0, y_1 \in \mathbb{R}$, explain why all polynomials p of degree ≤ 3 fulfilling

$$p\left(-\frac{1}{\sqrt{2}}\right) = y_0 \qquad , \qquad p\left(\frac{1}{\sqrt{2}}\right) = y_1$$

give the same value for the integral

$$\int_{-1}^{1} p(x)\frac{1}{\sqrt{1-x^2}}dx\,.$$

Calculate this value in dependence of y_0 and y_1.

Exercise 3

Compute the following integral

$$\int_{-\infty}^{+\infty} \cos(x)e^{-\frac{x^2}{2}}dx$$

using Monte Carlo quadrature (the Matlab function `randn` may be of help). Let N be the number of points used. Compute the exact value of the integral and plot the error of the Monte Carlo quadrature as a function of N. Use the composite Simpson rule to compute the integral numerically within an error of 10^{-6}. Can you achieve such precision when implementing Monte Carlo quadrature on your laptop?