Numerical Programming 1 (CSE) 2014

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Worksheet 4

Exercise 1

Use the matlab file Exercise3b.m posted together with worksheet 3 as follows: modify the file changing the function $f(x)$ from $f(x) = \sin(2\pi x)$ to

$$f(x) = \begin{cases} 2 & \text{if} \quad 0.25 < x < 0.75 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The Matlab command is `f=@(x) 2.0*(x>0.25 & x<0.75)`. Plot the trigonometric interpolation of $f(x)$ obtained with `DFT` (remember to set the variable `optioninterp` equal to `OptionB` in the code) for an increasing number of nodes N (with N even). Is the interpolation converging to the function f as you increase N? Does your observation contradict the error estimates we derived in class? J.W. Gibbs was the first one to give an explanation of the phenomenon you are observing.

Exercise 2

In the interval $[x_i, x_{i+1}]$ the cubic spline has the form

$$c_i(x) = a_i + b_i(x - x_i) + \frac{s_i}{6h_i}(x_{i+1} - x)^3 + \frac{s_{i+1}}{6h_i}(x - x_i)^3 \tag{2}$$

where $s_i = c_i''(x_i)$ and

$$a_i = f_i - \frac{s_i h_i^2}{6} \qquad \text{and} \qquad b_i = d_i - \frac{(s_{i+1} - s_i)h_i}{6}$$

with

$$d_i = \frac{f_{i+1} - f_i}{h_i}.$$

The second derivative s_i satisfies the recurrence relation

$$\frac{h_i}{6}s_i + \frac{h_i + h_{i+1}}{3}s_{i+1} + \frac{h_{i+1}}{6}s_{i+2} = d_{i+1} - d_i \qquad \text{with } i = 1, ..., n-1. \tag{3}$$

Consider again the Runge function $f(x) = \frac{1}{1+x^2}$. Compute the cubic spline interpolation by solving the tridiagonal system of equation (3) using the boundary condition $s_0 = s_n = 0.5$. Plot the interpolation together with the function $f(x)$.

Compute the integral from -1 to 1 of $f(x)$ by approximating the function with cubic spline above and integrating the spline interpolant. Do you get the result you were expecting as you increase n?

Exercise 3

Verify that the two point quadrature Gauss formula is exact for monomials up to degree three.