

Numerical Programming 1 (CSE) 2014

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Worksheet 3

Exercise 1

1. Given a T -periodic function f . Reformulate f as a 2π -periodic function \tilde{f} by linear transformation.
2. A measurement of the blood flow-rate in a cross-section of the carotid artery during a heart beat shows the following values:

time in ms	0	100	200	300	400	500	600	700	800	900
flow-rate	0	35	0.125	5	0	5	1	0.5	0.125	0

Use the FFT (the Matlab function `fft(y)` divided by the number of nodes n gives the coefficients c_j in the interpolating trigonometric polynomial) and polynomial interpolation to interpolate these data.

3. Plot your results and compare the two different kinds of interpolation.

Exercise 2

The Matlab file `Exercise3b.m` contains a code to interpolate the sine function $f(x) = \sin(2\pi x)$ using discrete Fourier transform (DSF)¹. Some of the lines of the code begin with the symbol `%`. Those lines represent comments and are not executed by Matlab. Read the code carefully and notice that, when setting the variable `optioninterp` to `OptionA`, the trigonometric interpolation does not match the function f , while when using the `OptionB` the graph of f and that one obtained using the DFT cannot be distinguished. Can you explain why this is the case?

Exercise 3

Consider equally spaced points $x_0, x_1, \dots, x_{N-1} \in [a, b]$. Let $f_i = f(x_i)$ and $f'_i = f'(x_i)$, where f' indicates the derivative of the smooth function $f : [a, b] \rightarrow \mathbb{R}$. One of the many possible uses of spline interpolation is to obtain approximation formulas to evaluate definite integrals. Consider

$$\int_{x_0}^{x_{N-1}} f(x) dx.$$

¹Download this file together with the auxiliary function `FT_bf.m` and store both files in the same folder.

Which approximation formula do you get by using a cubic spline interpolation for f and integrating the spline function?