Numerical Programming 1 (CSE) 2015

(C. Lasser, A. Schreiber and G. Trigila)

Worksheet 12

Review questions

- Write a Matlab code to compute the epsilon machine precision which does not use the Matlab command `eps`.
- Given a differentiable function $f(x) : \mathbb{R} \to \mathbb{R}$ define the condition number of f.
- Is the difference of two real positive numbers always well conditioned?
- Does the Newton method always converge?
- Given $N+1$ distinct points $\{(x_0, y_0), ..., (x_N, y_N)\}$, can you always find a unique polynomial $p(x)$ of degree less than or equal to N such that $p(x_i) = y_i$ for $i = 0, ..., N$?
- Consider an equispaced set of $N+1$ nodes in $[a, b]$. Does the algebraic interpolation error at any point of $[a, b]$ necessarily tend to 0 as N tends to infinity?
- Give the definition of Chebyshev points.
- Without using the `sqrt()` Matlab built-in function, write an algorithm to compute the square root of $x \in \mathbb{R}$. Estimate the rate of convergence or the operation count.
- What is the rate of convergence of the Monte Carlo quadrature as a function of the number N of sample points?
- Is an algorithm to compute the eigenvalues of an arbitrary matrix necessarily an iterative algorithm?
- Give an example in which the QR algorithm without shift does not converge.
- Is it true that the order of convergence of the trigonometric interpolation is higher when the points are sampled from a smooth function?
- Given the Rayleigh quotient $r(x) = \frac{x^T A x}{x^T x}$ (with $x \neq 0$) of a symmetric matrix $A = A^T \in \mathbb{R}^{n \times n}$, prove that its gradient satisfies $\nabla_x r(x) = 0$ if and only if $x \neq 0$ is an eigenvector of A.
- Given $A \in \mathbb{R}^{n \times n}$, how many flops are required to perform Gaussian elimination on A?

- Given $U \in \mathbb{R}^{n \times n}$ upper triangular, how many flops are required to solve the system $Ux = b$ via backward substitution?
- Without using the `det()` Matlab built-in function, write an algorithm to compute the determinant of $A \in \mathbb{R}^{n \times n}$. Give an estimate of the rate of convergence or the operation count.

Linear algebra background

- Can a matrix $A \in \mathbb{R}^{n \times n}$ always be diagonalized?
- Given a lower triangular matrix $L \in \mathbb{R}^{n \times n}$, what do the entries on the main diagonal represent?
- Prove that two similar matrices have the same eigenvalues. Do they also have the same eigenvectors?
- Given $A, B \in \mathbb{R}^{n \times n}$ diagonalizable, prove that if $AB = BA$, then there exists an invertible $Q \in \mathbb{R}^{n \times n}$ such that $A = Q^{-1}\Lambda_1 Q$ and $B = Q^{-1}\Lambda_2 Q$ with Λ_1 and Λ_2 diagonal matrices.
- What does it mean that in a finite dimensional vector space all the norms are equivalent?