Numerical Programming 1 (CSE) 2015

(C. Lasser, A. Schreiber and G. Trigila)

Worksheet 11

Exercise 1

- Compute by hand the SVD of the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -3 & 0 & -1 \end{bmatrix}.$$

- Compute by hand the singular values of the matrix

$$B = \begin{bmatrix} 9 & -2 & -1 & -6 \\ -2 & 7 & -3 & -2 \\ -1 & -3 & 6 & -2 \\ -6 & -2 & -2 & 10 \end{bmatrix}.$$

(hint: the eigenvalues of B are $\{15.59, 9.67, 6.74, 0\}$.)

- Given an invertible matrix $A \in \mathbb{C}^{n\times n}$, prove that $\kappa(A) = \|A\|_2\|A^{-1}\|_2 = \frac{\sigma_1}{\sigma_n}$ where $\sigma_1 \geq ... \geq \sigma_n$ are the singular values of A. Given the meaning of the singular values of a matrix A, what is the geometrical meaning of $\kappa(A)$?

Exercise 2

Compute the exact solution of the system of equations

$$\begin{cases} x'(t) &= -y(t) \\ y'(t) &= x(t) \end{cases}$$

with initial conditions $x(0) = 1$ and $y(0) = 0$ (hint: the Wikipedia page on simple harmonic oscillator may be of help). Solve numerically such system using forward and backward Euler discretization schemes. Plot the numerical solution on the $(x(t), y(t))$ plane for different values of time t. Explain the different behaviors obtained when using the two schemes in relation with the exact solution.

Exercise 3

- The trapezoidal scheme for solving $y'(t) = f(y, t)$ reads:

$$y_{n+1} = y_h + h\left(\frac{f(y_n, t_n) + f(y_{n+1}, t_{n+1})}{2}\right),$$

where $h > 0$ is the time step. What is the order of accuracy of this discretization scheme?

- Implement the trapezoidal rule to solve exercise number 2. Which differences do you notice with respect to the forward and backward Euler methods?