Numerical Programming 1 (CSE) 2015

(C. Lasser, A. Schreiber and G. Trigila)

Worksheet 10

Exercise 1

- Compute the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.$$

  Compute the first three steps of the power method $u^{k+1} = Au^k$ using $u_0 = [1, 0]^T$ as the initial (column) vector. What is the limit $\lim_{k\to\infty} u^k$?

- In general, by which quantity is the convergence of the power method characterized? Give an example of a matrix M for which the power method does not converge.

Exercise 2

The Rayleigh quotient iteration does the same as the inverse iteration with shift, except that the shift is given by the Rayleigh quotient and changes which each iteration step:
Starting with an initial vector $u_0 \in \mathbb{C}^n$ with $\|u_0\| = 1$ iterate for $k = 0, 1, 2, \ldots$:

- Compute the Rayleigh quotient $\mu_k = \langle u_k, Au_k \rangle$.
- Solve $(A - \mu_k Id)v_{k+1} = u_k$.
- Normalize $u_{k+1} = \frac{v_{k+1}}{\|v_{k+1}\|}$.

Implement the Rayleigh quotient iteration for the matrix A above and starting vector $u_0 = e_1$. Can you guess the convergence rate?

Exercise 3

- Compute one step of the QR method without shift (corresponding to $\kappa = 0$ in the notes) for finding the eigenvalues of

$$A = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & 0 \end{bmatrix}.$$

  Show that, after the first iteration, the off-diagonal entries of the matrix change from $\sin(\theta)$ to $-\sin(\theta)^3$. What does this fact imply with regard to the convergence of the algorithm?

- What are the eigenvalues of the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}?$$

  Can the QR method without shift be used to find the eigenvalues of A?