Numerical Programming 1 (CSE) 2014

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Worksheet 1

Exercise 1

Let `fl(x)` be the representative of $x \in \mathbb{R}$ in the set $\mathbb{F}$ of floating point numbers. In addition, let $a \oplus b = fl(a+b)$. Show that there exist a, b and c such that $(a \oplus b) \oplus c \neq a \oplus (b \oplus c)$. Assume 4 digits precision ($p = 4$ using the notation on the lecture notes) and let $a_1, ..., a_N \in \mathbb{R}$. How would you compute $S = \sum_{n=1}^{N} a_n$ so to keep the round off error small? Illustrate your point on a simple example (eg. taking $N = 3$) or writing a matlab function.

Exercise 2

- Let $f(x) = \frac{\ln(x)}{x}$, $x \neq 0$. Show that $f(x)$ is ill conditioned for x close to 1.
- Let $f(x) = 1 - \sin(x)$. For x close to $\bar{x} = \frac{\pi}{2}$ there is a cancellation problem. Can you manipulate $f(x)$ in order to obtain a better expression of this function (that avoids the cancellation problem for x close to $\bar{x}$)?
- Do the same thing with the function $h(x) = \frac{x}{1+x} - 1$ for $|\bar{x}|$ very big.

Exercise 3

Consider the polynomial

$$\begin{aligned} p(x) &= (x-2)^9 \\ &= x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 + \\ &\quad - 4608x^2 + 2304x - 512\,. \end{aligned}$$

1. Plot $p(x)$ for $x = 1.920, 1.921, \ldots, 2.080$, evaluating p via its coefficients $1, -18, 144, \ldots$. Use the Matlab functions `linspace(a,b,n)` and `polyval(p,x)`.
2. Produce the same plot again, now evaluating $p(x)$ via the expression $(x-2)^9$.
3. Compare your results and explain the differences.