MOCK EXAM FOR NUMERICAL PROGRAMMING I

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1. Basics: Matlab routines

Please provide short explaining sentences in the style of Matlab's help for the following built-in routines:

(1) `eps`
(2) `fft(x)`
(3) `trapz(y)`
(4) `randn`
(5) `qr(A) %A is a real mxn matrix`

Date: December 23, 2014, examination time is 90 mins.

2. Basics: Numerical tools

What is ...? Please provide one to two explaining sentences for each topic.

(1) cubic spline
(2) Gaussian quadrature
(3) condition number of an invertible matrix

3. Understanding

Please say yes or no, and provide one short explanation for your judgement.

(1) Gradual underflow occurs as soon as we work below machine precsion.
(2) Algebraic interpolation on $n = 100$ equidistant nodes is highly accurate.
(3) The summed trapezoidal rule has some connection to spline interpolation.
(4) Back substitution for a $n \times n$ system requires $O(n^3)$ flops.
(5) The conjugate gradient method requires to solve a triangular system at each iteration step.

4. Numerical programs: reading

What do the following numerical programs compute?

(1)
```
n = 100; x = 1+randn(n,1); q = sum(x)/n;
```
(2)
```
for i=1:n, x(i) = b(i)/L(i,i);
b(i+1:n) = b(i+1:n) - x(i)*L(i+1:n,i); end
```
(3)
```
a = A(:,1); v = a/norm(a) + sign(a(1))*eye(n,1);
H = eye(n) - 2*v*v'/norm(v)^2; B = H*A;
```
(4)
```
x = (A'*A)\(A'*b);
```
(5)
```
D = diag(diag(A)); L = -tril(A,-1); U = triu(A,1);
for k=1:m, x = D\b + D\(L*U)*x; end
```

5. Numerical programs: writing

(1) Please provide the $(k+1)$st step of a Newton iteration.
(2) Please write a MATLAB program performing n Newton iterations, starting in $x_0 = 2$ for searching a zero of $f(x) = \sin(x)$.
(3) Please explain shortly how to use the QR decomposition for solving least squares problems.
(4) Please write a MATLAB program solving a least squares problem via the QR decomposition.