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Exercises 1 and 2 are to be handed in on Wednesday, 20.06.2012, before the tutorial.

Exercise 1 (Weak Formulation of a PDE)

Determine a weak formulation of the following PDE

$$\begin{cases} -\frac{d}{dx}\left(a(x)u'(x)\right) + b(x)u(x) = f(x) & \text{in } (a,b), \\ u(a) = \alpha,\ u(b) = \beta. \end{cases}$$

Exercise 2 (Piecewise Linear Finite Elements)

Consider the PDE

$$\begin{cases} -u''(x) + u(x) = x & \text{in } (a,b), \\ u(a) = \alpha,\ u(b) = \beta. \end{cases}$$

and let $x_j = a + jh,\ j = 0, \dots, N+1$, with $h = \frac{b-a}{N+1},\ N \in \mathbb{N}$.

(a) Let $[a,b] = [0,1]$, $\alpha = 1$, $\beta = 2$. Use the approximation space

$$V_h = \left\{ v \in C[a,b] \ : \ v\big|_{[x_j, x_{j+1}]} \text{ linear } \forall j = 0, \dots, N \right\}$$

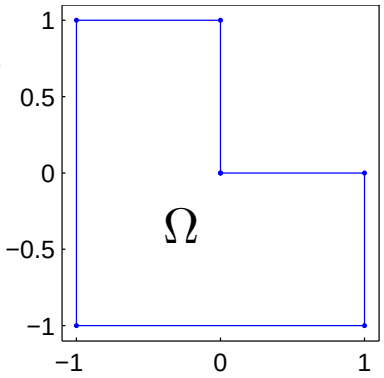
and its basis of shape functions $\varphi_0, \dots, \varphi_{N+1}$ defined by $\varphi_i(x_j) = \delta_{ij}$ to construct a linear system for the finite element approximation of the upper PDE.

(b) Compute the solution for different $N \in \mathbb{N}$ and estimate the convergence rate of your method. Does it match your expectations?

Exercise 3 (`pdetool`)

On an L-shaped domain Ω defined by the points $(0,0), (1,0), (1,-1), (-1,-1), (-1,1), (0,1), (0,0)$ consider the following PDE:

$$\begin{cases} -\nabla\left[(2+xy)\nabla u\right] + 3u(x) = 400xy & \text{in } \Omega, \\ u(x,y) = \sin(2\pi xy) & \text{for } x = 0 \text{ or } y = 0, \\ u(x,y) = 0 & \text{for } x = 1 \text{ or } y = 1, \\ u(x,y) = \sin(\pi xy) & \text{for } x = -1 \text{ or } y = -1. \end{cases}$$



Use the Matlab GUI `pdetool` to plot an approximation to its solution.