Caroline Lasser, Ilja Klebanov
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The exercises are to be handed in on Wednesday, 11.07.2012, before the tutorial.

Exercise 1 (Stability of AB2)

Assume the heat equation is solved by applying the 2-stage Adams-Bashforth-method in time and central differences in space.

(a) Derive the following condition for the stability of the method:

$$\left|1 - 6\mu s \pm \sqrt{(1-6\mu s)^2 + 8\mu s}\right| \le 2 \qquad \forall s \in [0,1].$$

Hint: $e^{i\varphi} - 2 + e^{-i\varphi} = -4\sin^2(\varphi/2)$.

(b) Choose $\mu_1, \mu_2 > 0$, such that the method is stable for μ_1 and unstable for μ_2.

Exercise 2 (Crank-Nicolson for the Advection Equation)

Assume the advection equation is solved by applying the trapezoidal rule in time and discretizing the space derivative by $\partial_x u \approx \frac{1}{\Delta x}\mathcal{Y}_0\Delta_0 u$.

(a) Formulate the method (in terms of $u_k^j = u(x_k, t_j)$) and the resulting linear system in matrix notation.

(b) Discuss the stability of the method.

Exercise 3 (Upwind Euler Method for the Advection Equation)

Consider the forward Euler Method is applied to the advection equation in time and the space derivative is discretized by $\partial_x \approx \frac{1}{\Delta x}\Delta_-$.

(a) Implement the resulting method on $x, t \in [0,1]$ with initial and boundary conditions

$$u(x,0) = \cos(8\pi x)\,, \qquad u(0,t) = \cos(4\pi t).$$

For which $\mu = \frac{\Delta t}{\Delta x}$ do you observe stable behaviour?

(b) Find the analytical solution and estimate the convergence rate by applying your program to different Δx (choose $\mu = \frac{1}{2}$).