Caroline Lasser, Ilja Klebanov
22th November 2011

Exercises 2 and 3 are to be handed in on Tuesday, 29.11.2011, before the lecture.

Exercise 1 (Polynomial Interpolation)

(a) Interpolate the function $f(x) = sin(2\pi x)$ on the interval $[-1, 1]$ by considering 20 equally spaced nodes $x_1, \ldots, x_{20}$ (use polynomial interpolation).

(b) Now, multiply $y_{10} = f(x_{10})$ by $(1 + 10^{-2})$ and interpolate this perturbed data again.

(c) Plot your results and compare them.

The Matlab functions `polyfit` and `polyval` might be of help.

Exercise 2 (Chebyshev interpolation)

(a) Use linear transformation to scale the Chebyshev nodes from the interval $[-1, 1]$ to an arbitrary interval $[a, b]$.

(b) Consider Runge's function $f(x) = \frac{1}{1+x^2}$ on the interval $[-5, 5]$.
Compare the interpolation error (in the $\|.\|_\infty$-norm) of equally spaced nodes to Chebyshev nodes (scaled to $[-5, 5]$, see (a)) for $n = 5,\ 10,\ 20,\ 40$, where n denotes the number of nodes (use Matlab).
You may approximate the $\|.\|_\infty$-norm of a function g by
$\|g\|_\infty \approx max\{|g(a_i)| : a_i = -5 + \frac{i}{100},\ i = 0, \ldots, 1000\}$.

Exercise 3 (FFT)

(a) Given a T-periodic function f. Reformulate f as a 2π-periodic function $\tilde{f}$ by linear transformation.

(b) A measurement of the blood flow-rate in a cross-section of the carotid artery during a heart beat shows the following values:

time in ms	0	100	200	300	400	500	600	700	800	900
flow-rate	0	35	0.125	5	0	5	1	0.5	0.125	0

Use the FFT to interpolate these data (the Matlab function `fft(y)` devided by the number of nodes n gives the coefficients c_j in the interpolating trigonometric polynomial).

(c) Plot your results and compare them to polynomial interpolation.