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Exercises 1(c) , 2, 3 (a) and(b) are to be handed in on Thursday, 22.11.2011, before the lecture.

Exercise 1 (Bisection method)

Assume that a continuous function $f : \mathbb{R} \to \mathbb{R}$ has a unique zero in the interval (a, b) and that $f(a)f(b) < 0$ and let a number $n \in \mathbb{N}$ be given. Look at the following pseudo-code:

```
for k = 1,...,n
    x = (a+b)/2
    display a, b
    if f(x) = 0
        return x
    else if f(x)f(a) < 0
        b = x
    else if f(x)f(b) < 0
        a = x
    end
end
return x
```

(a) What does this algorithm do and why is it called „bisection method"?

(b) Implement this algorithm (`bisec(f,a,b,n)`) and test it for $f(x) = x^2 - 1,\ a = 0,\ b = 3$.

(c) In addition, plot the error of each iteration step over n and estimate the convergence rate (a semilogarithmic plot might be appropriate here - Matlab: `semilogy(x,y)`). Compare this result with the one of Newton's method (see exercise 2 on the 3rd exercise sheet, function f_2).

Exercise 2 (Newton's Method)

Use Newton's Method to compute the zero of $f(x) = -\frac{2}{3}2^{-x} + 4^{-x} + \frac{1}{9}$.
Plot the error of each iteration step over n and estimate the convergence rate (the exact result is given by $x^* = \frac{log(3)}{log(2)}$, use a semilogarithmic plot again). Why is it not quadratic?

Exercise 3 (Multidimensional Newton's Method)

Consider the (nonlinear) function $F : \mathbb{R}^2 \to \mathbb{R}^2,\ \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x^2 - 3xy \\ 5xy \end{pmatrix}$.

(a) Calculate the zeros of F analytically.

(b) Compute the Jocobian matrix $J(x, y)$.

(c) Implement a Matlab function `newtF(x_0,y_0,n)` which performs and displays n Newton iteration steps for the computation of a zero $(x, y) \in \mathbb{R}^2$ of F starting in (x_0, y_0) and test it for $(x_0, y_0) = (3, 4)$.