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Exercises 1 and 2 are to be handed in on Tuesday, 15.11.2010, before the lecture.

Exercise 1 (Backward Stability) ($*$)
Prove that the multiplication of two real numbers $\left(f : \mathbb{R}^2 \to \mathbb{R},\ (x,y) \mapsto xy\right)$ computed by $\tilde{f}(x,y) = fl(x)\tilde{\cdot}fl(y)$ is backward stable ($\tilde{\cdot}$ denotes the floating point number multiplication).

Exercise 2 (Making Use of Error Propagation) ($*$)
Consider the integrals $\gamma_k = \int_0^1 t^{k-1}e^{1-t}dt,\ k \in \mathbb{N}$.

(a) Show that the numbers γ_k fulfill the *backward-recursion*

$$\gamma_k = \frac{1}{k}(\gamma_{k+1} + 1)\,,\ \ k \in \mathbb{N}.$$

(b) Starting with an index n and an estimated value $\tilde{\gamma}_n = \gamma_n + \Delta\gamma_n$ of γ_n , the upper recursion is applied repeatedly in order to compute $\tilde{\gamma}_m = \gamma_m + \Delta\gamma_m$, $m < n$. What will the error $\Delta\gamma_m$ be (do not work in floating point arithmetics)?

(c) For a given index $m \in \mathbb{N}$ find an index n **and** an estimated value $\tilde{\gamma}_n = \gamma_n + \Delta\gamma_n$, such that (after applying the recursion repeatedly, as described in (b)) the relative error $\frac{\Delta\gamma_m}{\gamma_m}$ of γ_m will be less than 10^{-16}. You may use the fact that $\gamma_k \underset{k\to\infty}{\searrow} 0$.
Implement the resulting algorithm!

Exercise 3 (Newton Method)
Write a Matlab function `newt(f,df,x_0,n)` (you can use the Matlab function `inline()` to use functions as parameters, e.g. `f = inline('atan(x)')`), that performs n iteration steps of the Newton method for the function f, starting in x_0 (df denotes the derivative of f)!
Test your function for $f_1(x) = atan(x),\ f_2(x) = x^2 - 1,\ f_3 = x^3 - 6x^2 + 10x - 2$ and $x_0 = 2$.
For which of these functions does the Newton method not converge? Explain graphically (by hand or using Matlab) what goes wrong in these cases.