Caroline Lasser, Ilja Klebanov
24th January 2012

The exercises are to be handed in on Tuesday, 31.01.2012, before the lecture.

Exercise 1 (Vector Iteration)

Let $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 0 & -1 & 1 \end{pmatrix}$.

Determine the eigenvalues of A by implementing the vector iteration and the inverse vector iteration with shift $\mu = 0$ and $\mu = 2$ (as initial vector choose e.g. $x_0 = e_1$).
Try to guess a good stop criterion and determine the convergence rates of the respective eigenvalues.

Exercise 2 (Rayleigh Quotient Iteration)

The Rayleigh quotient iteration does the same as the inverse iteration with shift, except that the shift is given by the rayleigh quotient and changes which each iteration step:
Starting with an initial vector $u_0 \in \mathbb{C}^n$ with $\|u_0\| = 1$ iterate for $k = 0, 1, 2, \dots$:

(i) Compute the Rayleigh quotient $\mu_k = \langle u_k, Au_k \rangle$.

(ii) Solve $(A - \mu_k Id)v_{k+1} = u_k$.

(iii) Normalize $u_{k+1} = \frac{v_{k+1}}{\|v_{k+1}\|}$.

Implement the Rayleigh quotient iteration for the matrix A above and starting vector $u_0 = e_1$. Can you guess the convergence rate?

Exercise 3 (QR Iteration)

Let Implement the QR-iteration (without shift) for the upper matrix (use 50 iteration steps):
Choose $A_0 = A$ and iterate for $k = 0, 1, 2, \dots$:

(i) Compute the QR-decomposition of A_k: $\quad A_k = Q_k R_k$.

(ii) Choose $A_{k+1} = R_k Q_k$.

Determine the convergence rates of the diagonal converging to the eigenvalues (you may use the Matlab function `eig(A)` for reference values) and of the sub- and superdiagonal converging to zero.