Caroline Lasser, Ilja Klebanov
25th October 2011

Exercises 1 and 2 are to be handed in on Thursday, 03.11.2011, before the lecture.

Exercise 1 (Floating Point Numbers) (∗)

Generalize the Matlab function `floating_point` presented in the lecture to a function `floating_point(b,p,emi,ema)` that calculates and plots the set of floating point numbers

$$\mathbb{F}(b,p,e_{min},e_{max}) = \left\{ \pm m b^{e-p} \mid b^{p-1} \le m \le b^p - 1,\ e_{min} \le e \le e_{max} \right\}$$

for arbitrary parameteres b, p, e_{min}, e_{max}.
(Use the Matlab-function `sort(a)` if you want to sort the vector a.)
Now let $b = 5,\ p = 1$. Calculate $fl(x)$ for $x = 12,\ x = 2.5$.
You may use your program, but present the result in the form $\pm m b^{e-p}$.
Determine the machine epsilon ϵ_M and compare it two the relative rounding errors of your results. Can you find an $x \in \mathbb{R}$ such that the relative error of $fl(x)$ is at least $\frac{1}{3}\epsilon_M$, $\frac{2}{3}\epsilon_M$ respectively?

Exercise 2 (Fibonacci Numbers) (∗)

The Fibonacci numbers f_n $(n \in \mathbb{N})$ are defined by

$$f_n = f_{n-1} + f_{n-2} \ \text{(for } n \ge 2),\quad f_0 = 0,\quad f_1 = 1.$$

Write the following Matlab functions:

1. `fib(n)`, the output of which is the list $[f_n, f_{n+1}]$ (e.g. `fib(6)` = `[8,13]`).
2. `invfib([a,b],n)`, which inverts the upper recursion n times, in particular `invfib(fib(n),n)` = `[0,1]` (e.g. `invfib([8,13],3)` = `[2,3]`, `invfib([8,13],6)` = `[0,1]`).

Test your functions for different input arguments. Calculate `invfib(fib(80),80)`. Why is it not $[0,1]$ as it should be?

Exercise 3 (Evaluating Polynomials)

Consider the polynomial

$$\begin{aligned} p(x) &= (x-2)^9 \\ &= x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512\,. \end{aligned}$$

(a) Plot $p(x)$ for $x = 1.920,\ 1.921,\ \ldots,\ 2.080$, evaluating p via its coefficients $1,\ -18,\ 144,\ \ldots$. Use the Matlab functions `linspace(a,b,n)` and `polyval(p,x)`.

(b) Produce the same plot again, now evaluating p via the expression $(x-2)^9$.

(c) Compare your results and explain the differences.