

Exercises 1 and 2 are to be handed in on Thursday, 03.11.2011, before the lecture.

Exercise 1 (Floating Point Numbers) (*)

Generalize the Matlab function `floating_point` presented in the lecture to a function `floating_point(b, p, emi, ema)` that calculates and plots the set of floating point numbers

$$\mathbb{F}(b, p, e_{min}, e_{max}) = \left\{ \pm mb^{e-p} \mid b^{p-1} \leq m \leq b^p - 1, e_{min} \leq e \leq e_{max} \right\}$$

for arbitrary parameters b, p, e_{min}, e_{max} .

(Use the Matlab-function `sort(a)` if you want to sort the vector a .)

Now let $b = 5$, $p = 1$. Calculate $fl(x)$ for $x = 12$, $x = 2.5$.

You may use your program, but present the result in the form $\pm mb^{e-p}$.

Determine the machine epsilon ϵ_M and compare it two the relative rounding errors of your results. Can you find an $x \in \mathbb{R}$ such that the relative error of $fl(x)$ is at least $\frac{1}{3}\epsilon_M$, $\frac{2}{3}\epsilon_M$ respectively?

Exercise 2 (Fibonacci Numbers) (*)

The Fibonacci numbers f_n ($n \in \mathbb{N}$) are defined by

$$f_n = f_{n-1} + f_{n-2} \quad (\text{for } n \geq 2), \quad f_0 = 0, \quad f_1 = 1.$$

Write the following Matlab functions:

1. `fib(n)`, the output of which is the list $[f_n, f_{n+1}]$ (e.g. `fib(6) = [8, 13]`).
2. `invfib([a, b], n)`, which inverts the upper recursion n times, in particular `invfib(fib(n), n) = [0, 1]` (e.g. `invfib([8, 13], 3) = [2, 3]`, `invfib([8, 13], 6) = [0, 1]`).

Test your functions for different input arguments. Calculate `invfib(fib(80), 80)`. Why is it not `[0, 1]` as it should be?

Exercise 3 (Evaluating Polynomials)

Consider the polynomial

$$\begin{aligned} p(x) &= (x - 2)^9 \\ &= x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512. \end{aligned}$$

- (a) Plot $p(x)$ for $x = 1.920, 1.921, \dots, 2.080$, evaluating p via its coefficients $1, -18, 144, \dots$. Use the Matlab functions `linspace(a, b, n)` and `polyval(p, x)`.
- (b) Produce the same plot again, now evaluating p via the expression $(x - 2)^9$.
- (c) Compare your results and explain the differences.