Triangular 2D and 3D Grid Refinement for Atmosphere and Ocean Simulation

Jörn Behrens

Technische Universität München
Center for Math. Sciences (M3)
85747 Garching, Germany

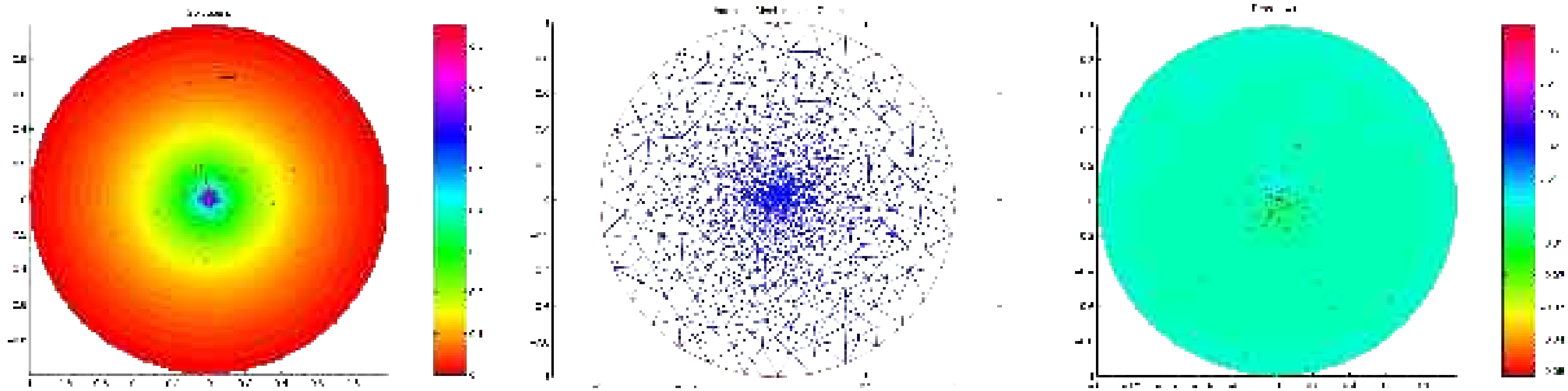
behrens@ma.tum.de
www-m3.ma.tum.de/m3/behrens

DEKLIM Projekt Nr. 01 LD 0037

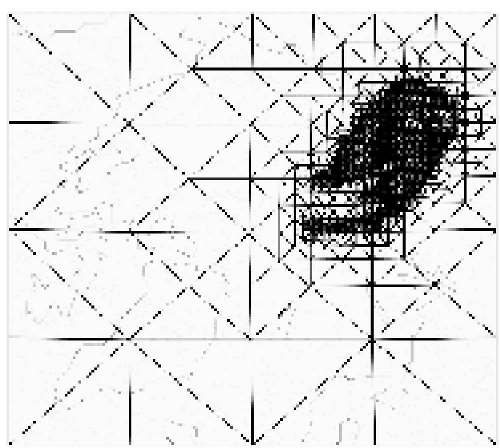
DFG-Stipendium Nr. BE2314/3-1

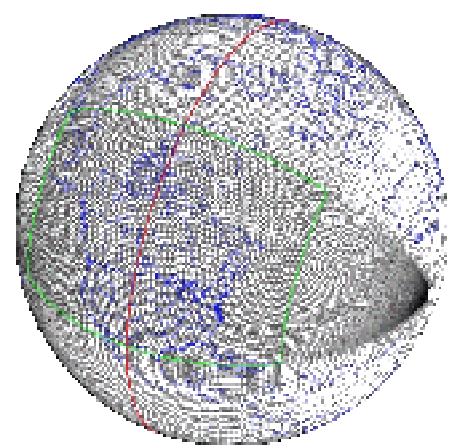
Introduction – Perspective Adaptivity

Equidistribution of error → **rigorous error notation**



Enhanced resolution → **scale analysis**





Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

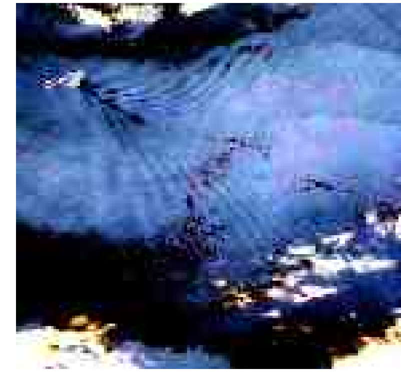
Introduction – why adaptive modeling?

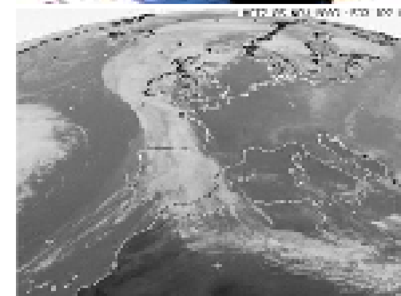
- Scale interaction
- Sensitivity analysis (local – global scale)

- Fronts (large gradients)
- Embedded local phenomena

- Filamentation in tracers
- Point sources for tracers

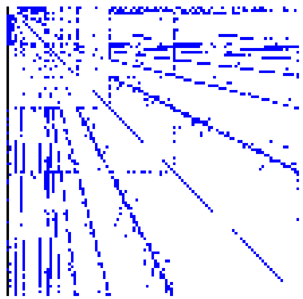
- Efficient utilization of computing resources

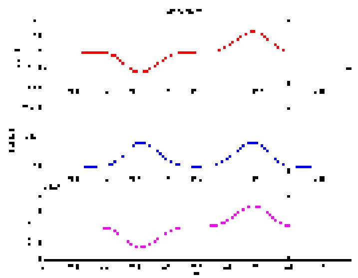




Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Modular Adaptive Software





System solver

Dynamic kernel (conserv. SLM)

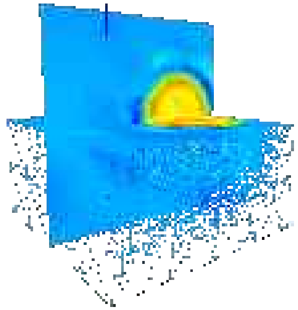
Sub-grid processes

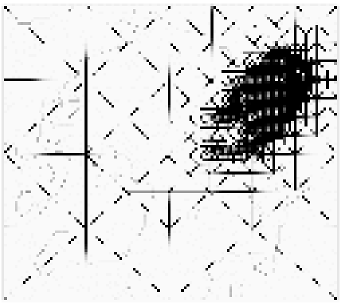
Diagnostics visualization

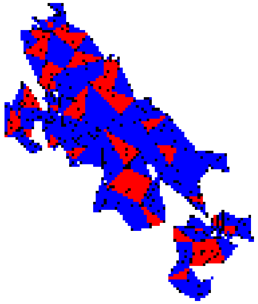
main

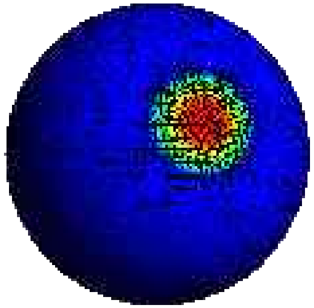
I/O, Data management

Grid generator amatos
(http://www-m3.ma.tum.de/software/amatos)



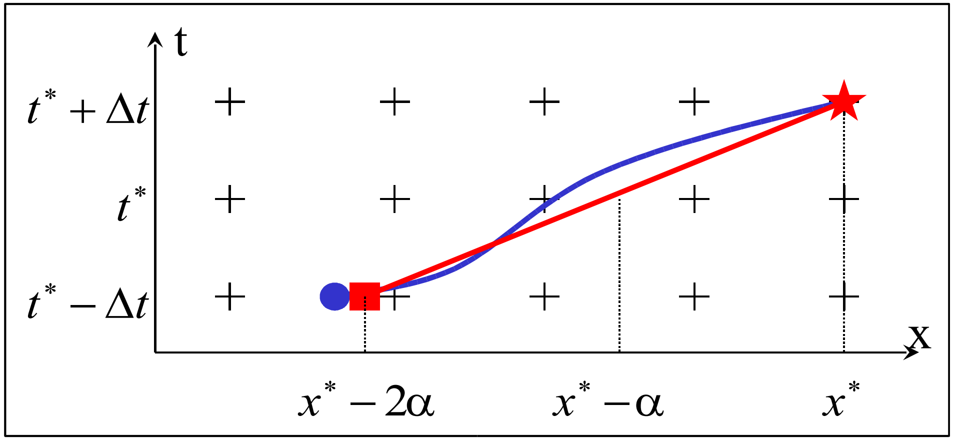






Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Discretization



$$0 = \frac{dc}{dt} \approx \frac{c(\vec{x}, t + \Delta t) - c(\vec{x} - 2\vec{\alpha}, t - \Delta t)}{2\Delta t}$$

$$\Rightarrow \quad c(\vec{x}, t + \Delta t) = c(\vec{x} - 2\vec{\alpha}, t - \Delta t)$$

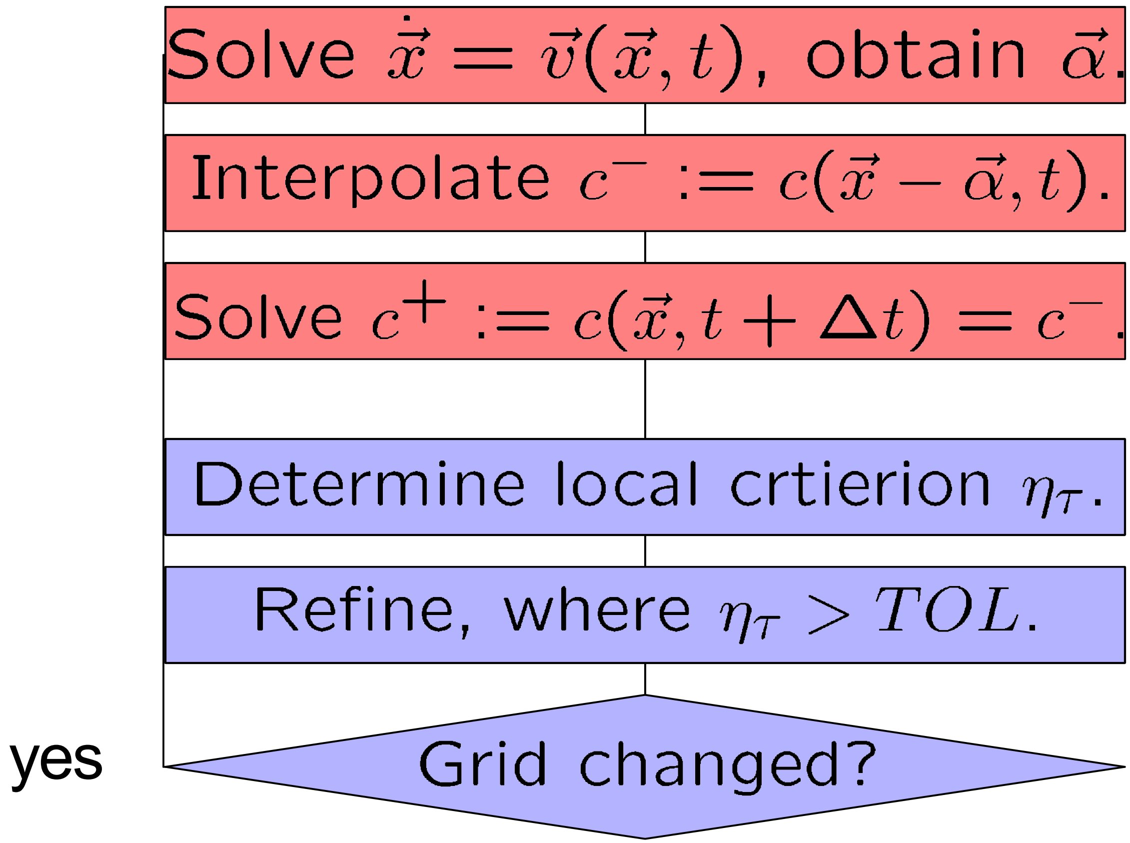
$$\frac{\vec{\alpha}}{\Delta t} \approx \frac{d\vec{x}}{dt} = \vec{v}(\vec{x}, t)$$

Iteration: $\vec{\alpha}^{k+1} = dt \cdot \vec{v}(\vec{x}^\star - \vec{\alpha}^k, t^\star)$

A. Wiin-Nielsen (1959)

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

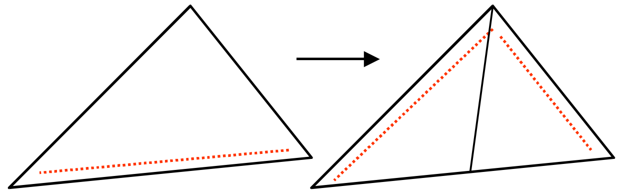
Algorithmus: Semi-Lagrange Methode (SLM)



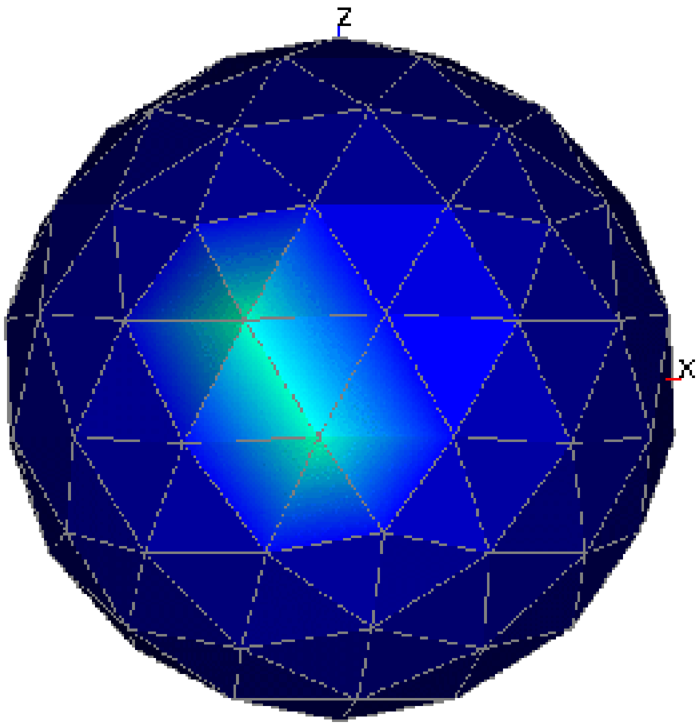
Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

J. B., (1996)/(1998)

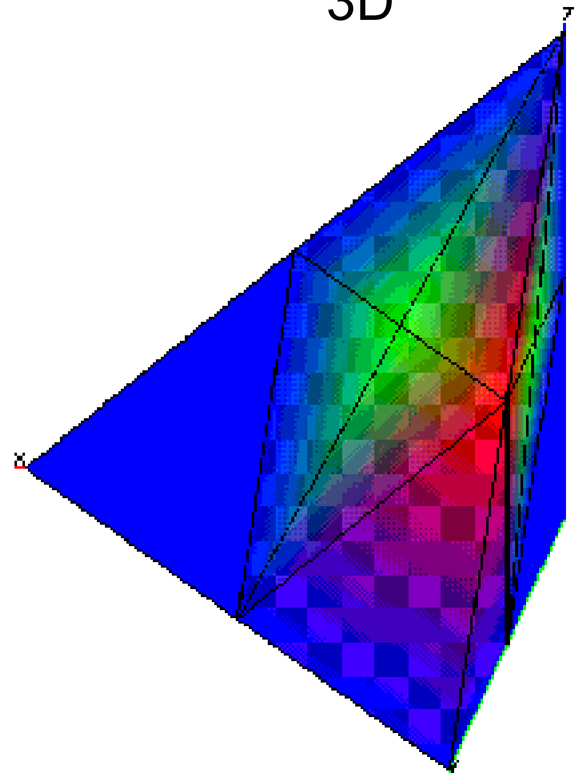
Refinement Strategy



2D



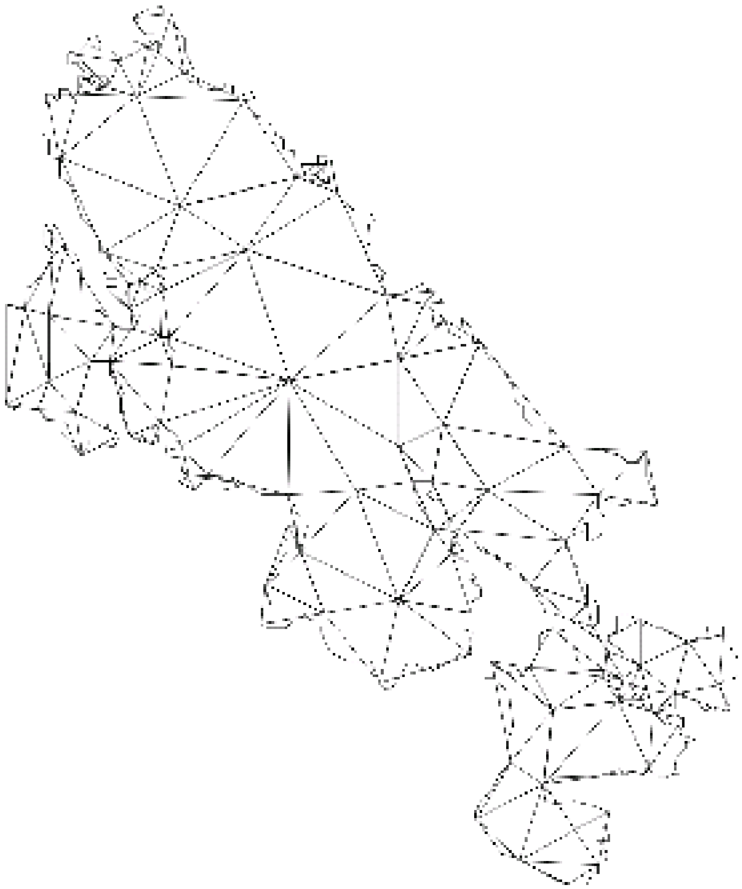
3D



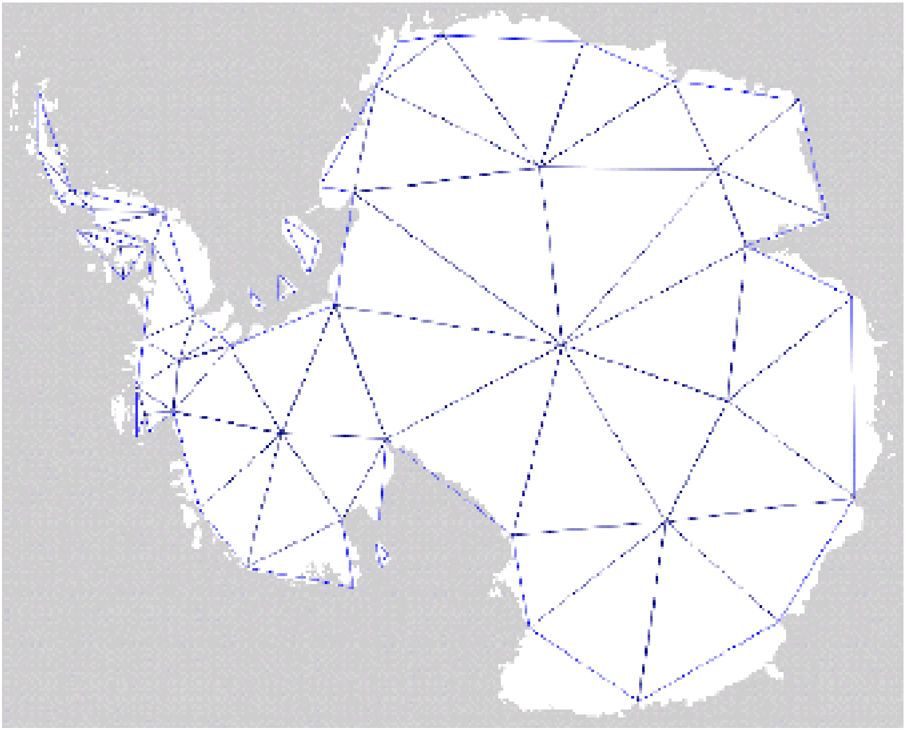
Rivara (1984), Bänsch (1991), Grids created with amatos

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Complex Geometries



Polygonal domain

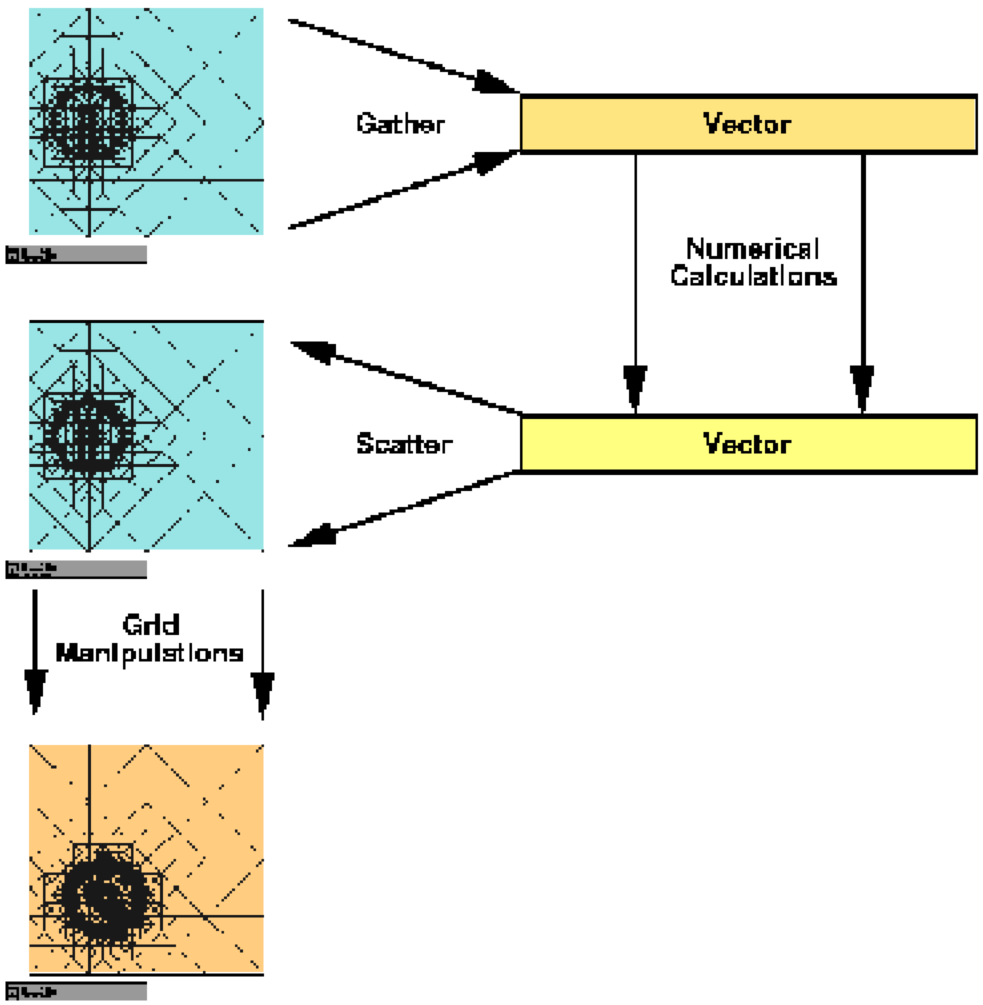


Bitmapped domain

Grid created by amatos (with F. Klaschka)

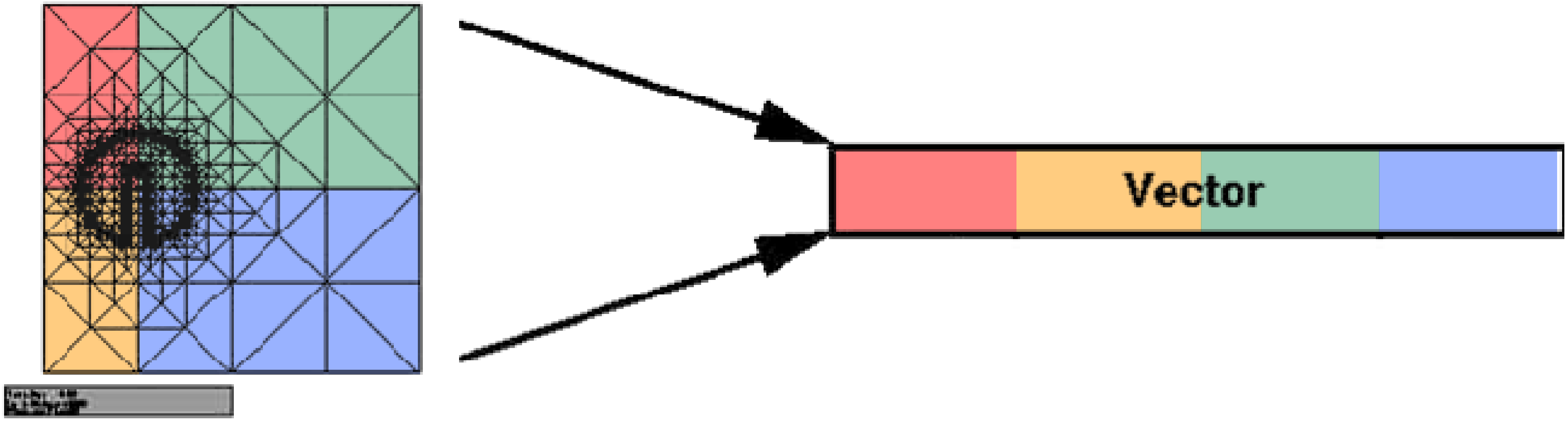
Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Data Management and Numerics



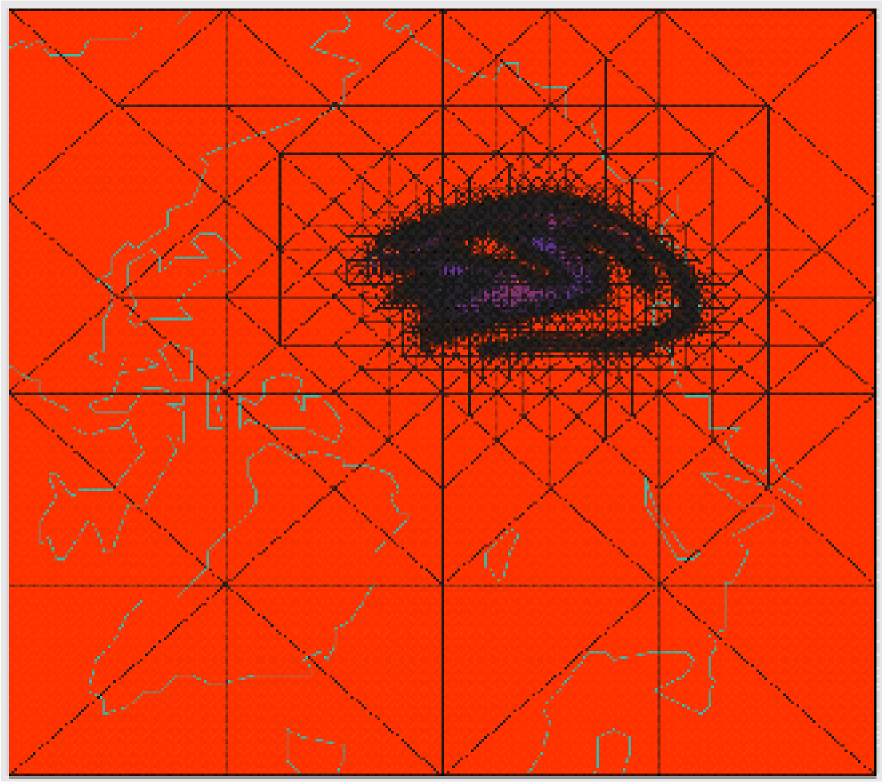
Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Data Management and Parallelization



Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Parallelization



Partitioning problem

- Distribute cells in equally sized sets (partitions)
- Partitions shall be connected
- Partitions have to be re-calculated frequently
- Data movement has to be minimized
- Algorithm has to be parallel/low computational effort

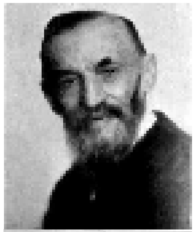
Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Parallelization

Excursion: Space-filling curves

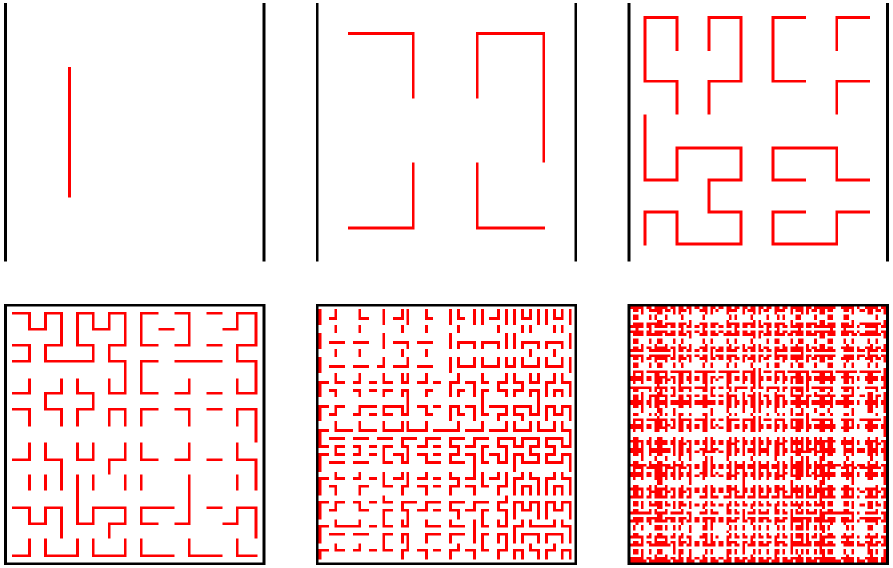
Question: $\exists$ surj. $\wedge$ cont. map $\mathbb{R} \rightarrow \mathbb{C}$?

Answer: Yes! Constructive recursive proof.



Giuseppe Peano
(1858-1932)

David Hilbert
(1862-1943)

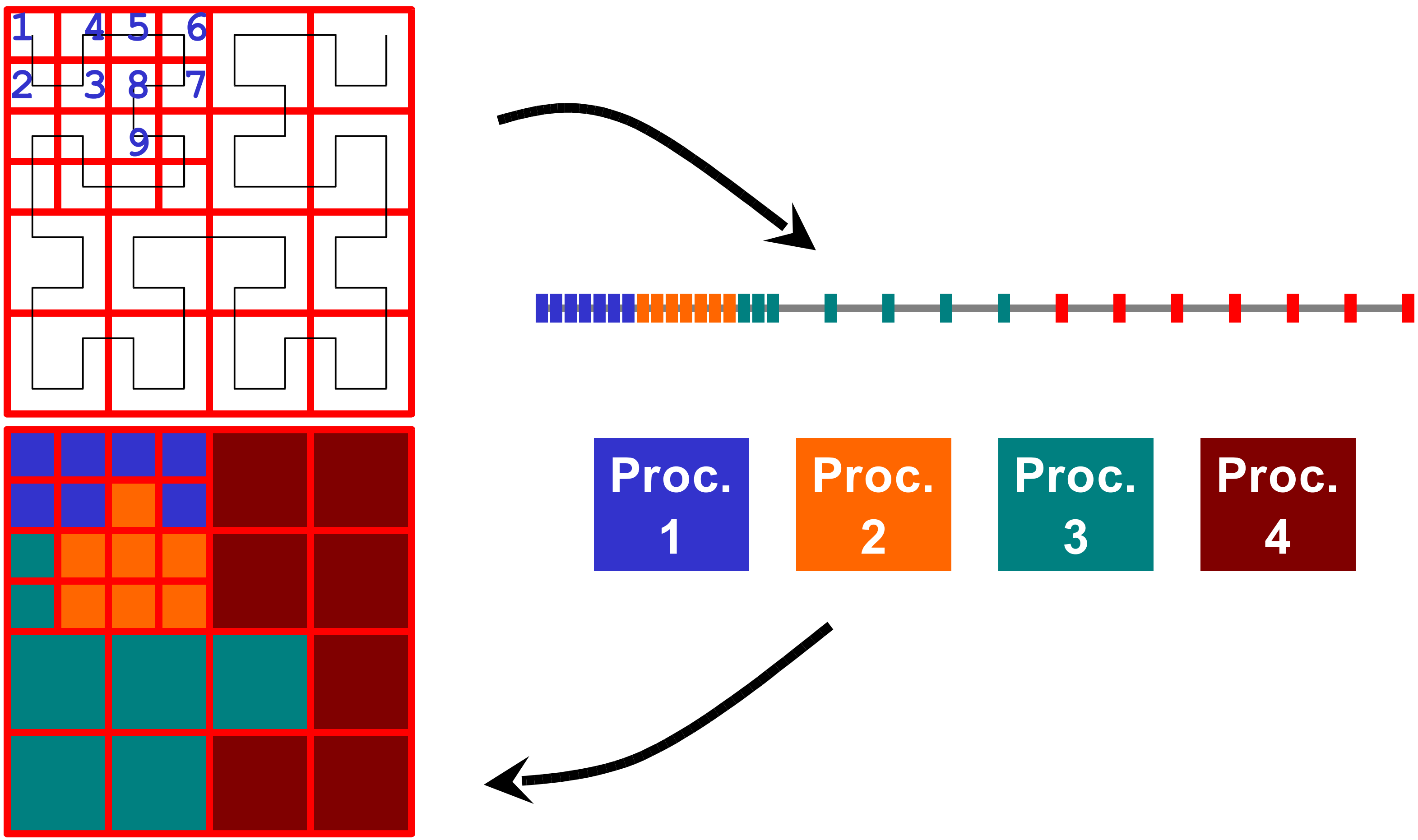


Peano, 1889; Hilbert, 1890

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Parallelization

Space-filling curve for load balancing



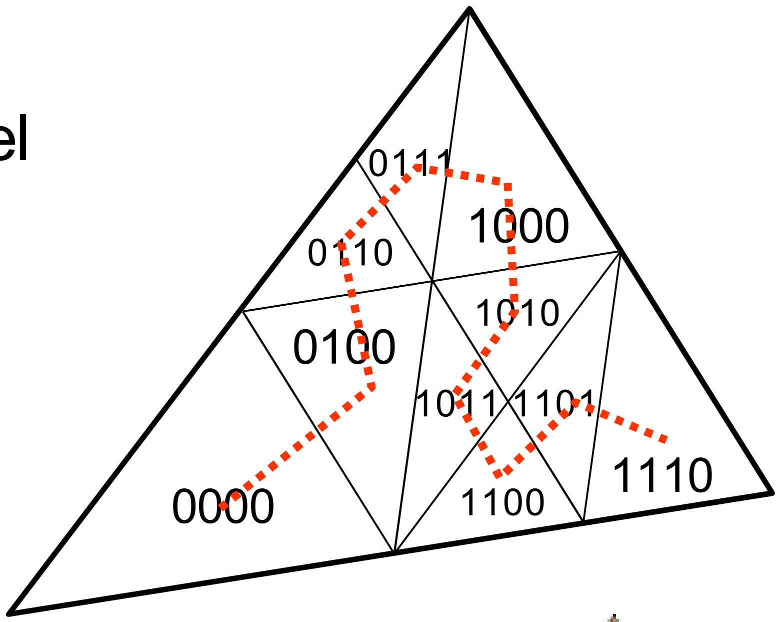
Roberts et al. 1997, Griebel & Zumbusch, 1999

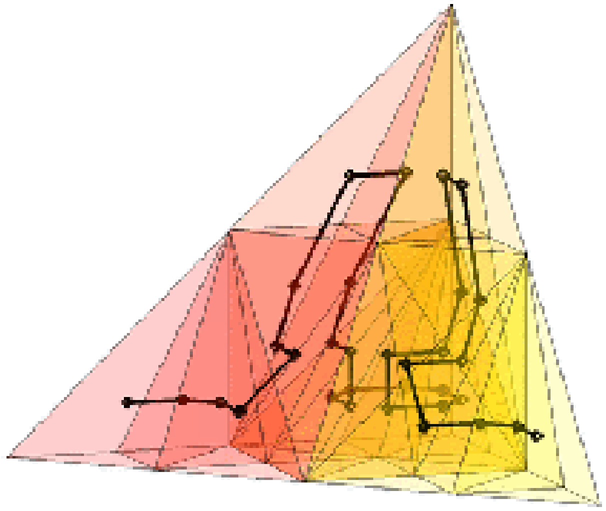
Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Parallelization

Algorithm for triangles

1. One bit per refinement level
2. Set bits while refining





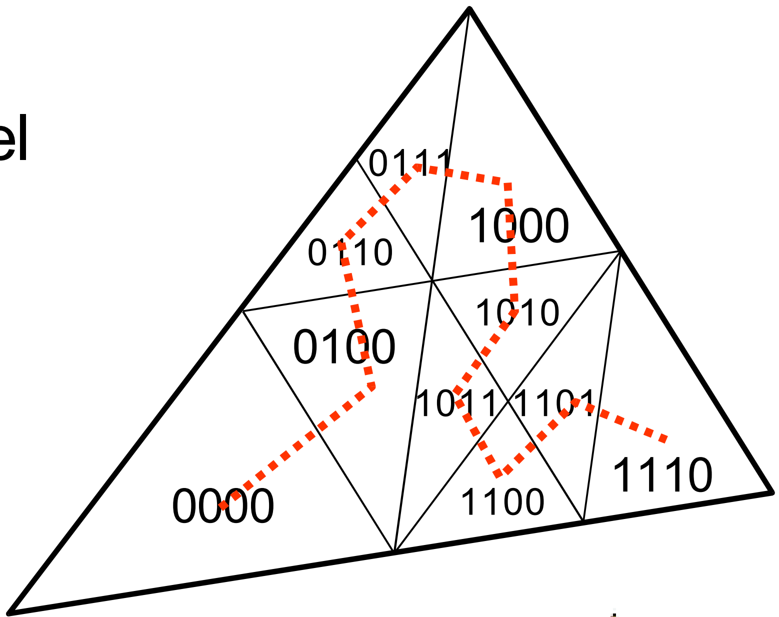
J. B., J. Zimmermann (2000), N. Rakowski (2003)

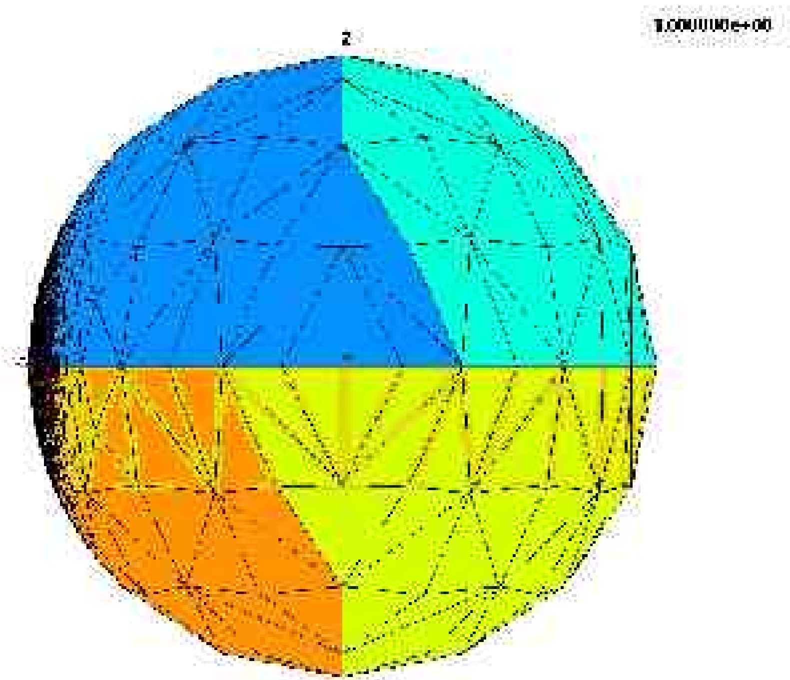
Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

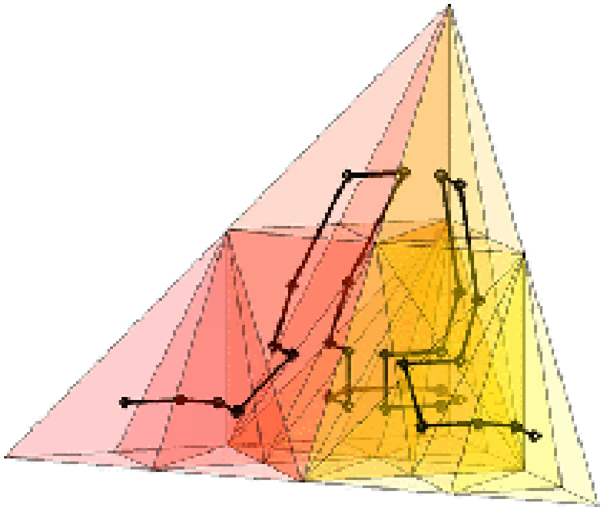
Parallelization

Algorithm for triangles

1. One bit per refinement level
2. Set bits while refining







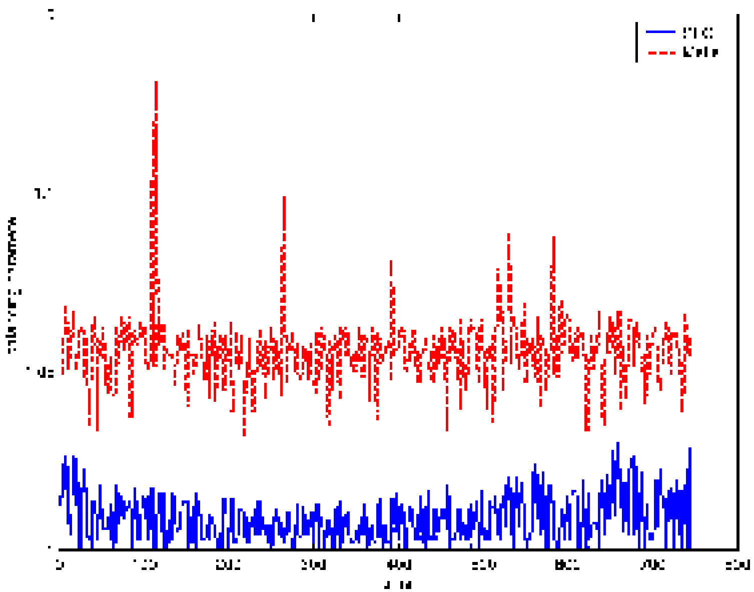
J. B., J. Zimmermann (2000), N. Rakowski (2003)

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

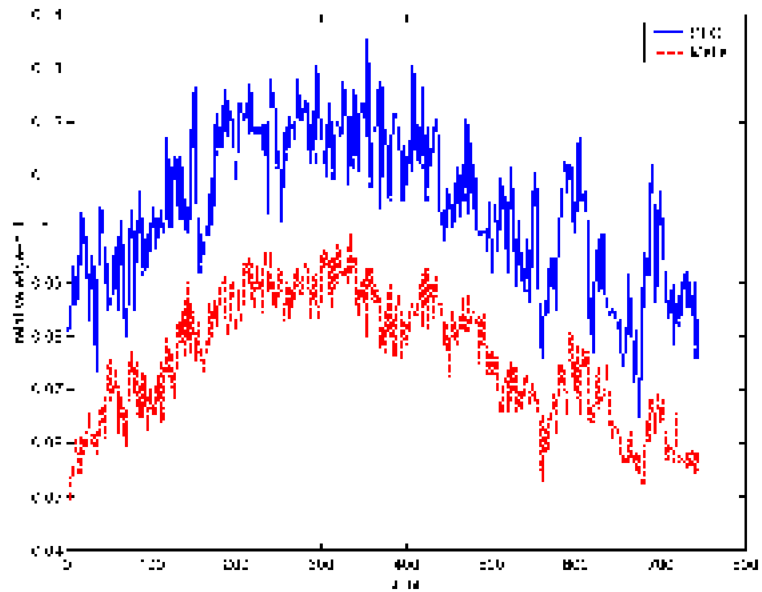
Results: Tracer Advection

Artificial tracer in Arctic stratosphere

Load balancing



Edge-cut

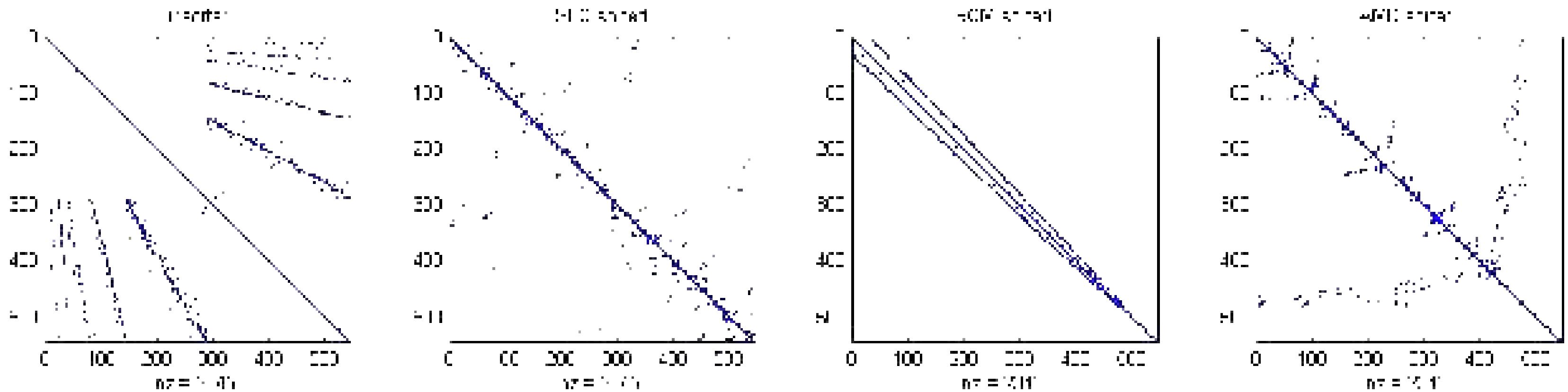


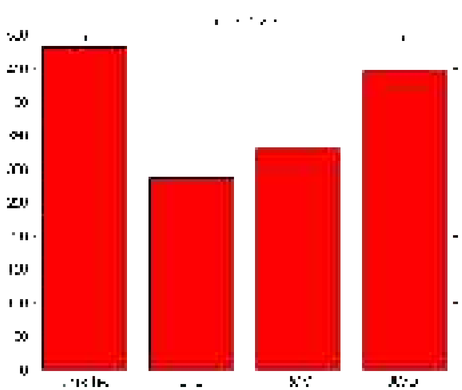
SFC: J. B., J. Zimmermann (2000), Metis: G. Karypis, V. Kumar (1998)

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

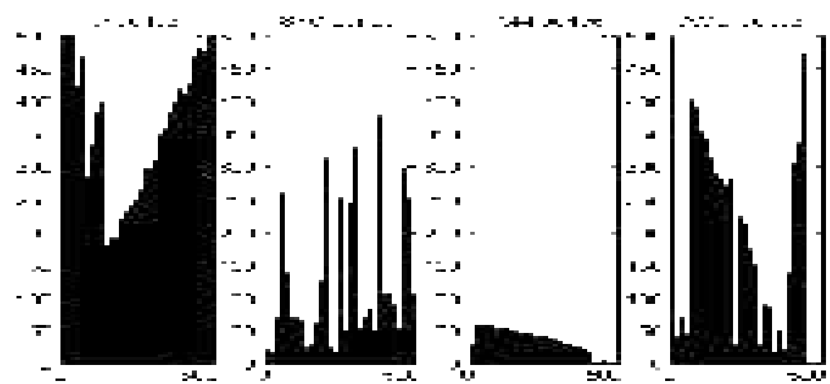
Data Management revisited

Connectivity matrix with different orderings





Cache misses

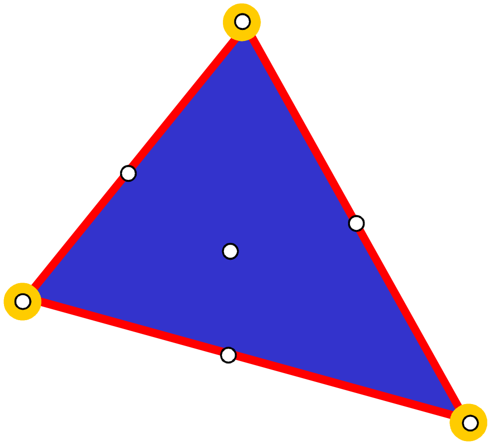


Distance structure

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

FEM support

Main data objects:
nodes, edges, triangles



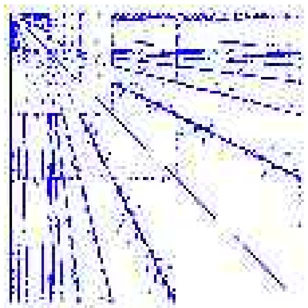
FEM-Signature:

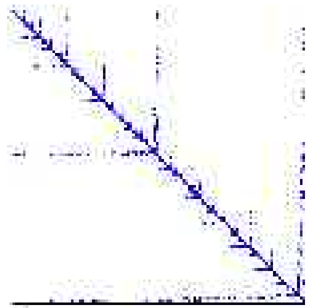
- Unknowns on nodes
- Unknowns on edges
- Unknowns on triangles
- Position in barycentric coordinates
  (for edges and triangles)

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

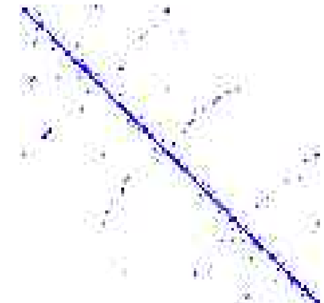
Space-Filling Curves: Matrix Ordering

Structure of matrix

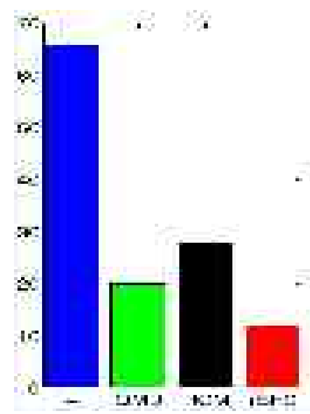

tree-sorted

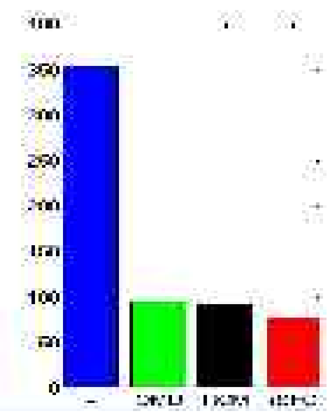

quotient minimum degree


reverse Cuthill-McKee


reverse SFC

- System with ~200.000 unknowns
- Utilizes preconditioned BiCGStab
- ILU pre-conditioning


Iterations


Time

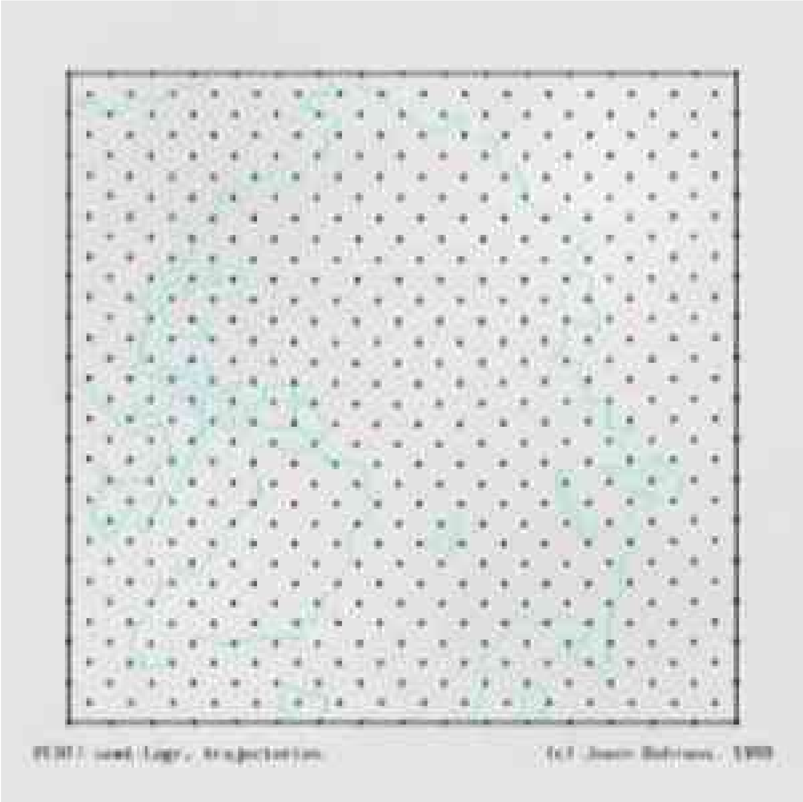
J. B., N. Rakowski, S. Frickenhaus, et al. (2003)

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Example 1: linear advection

Simulation of tracer transport

Resolution of wind data:
50 x 50 km
Situation in January 1990,
70 hPA layer (18.000 m)

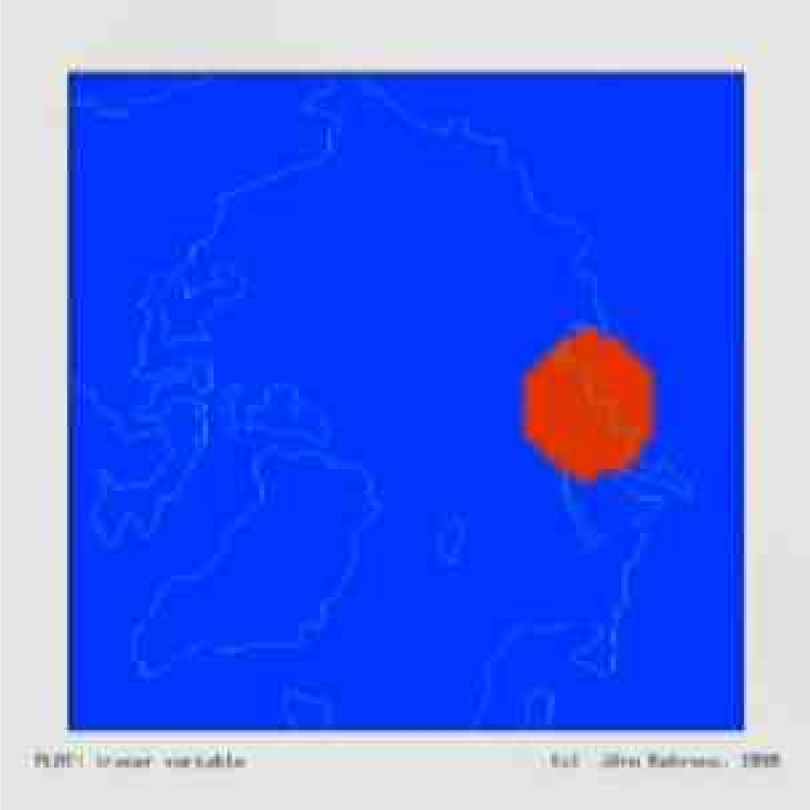


A. Rinke et al., 1997

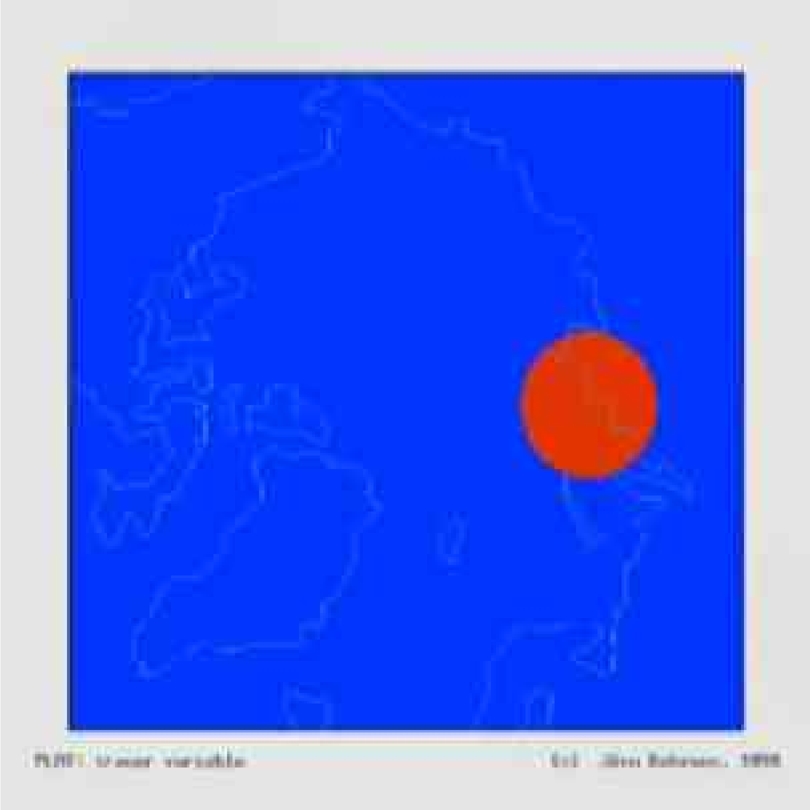
Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Example 1: linear advection

Simulation of tracer transport



Resolution: 50 km uniform



Resolution: 5 km local

J. B., K. Dethloff, W. Hiller, A. Rinke (2000)

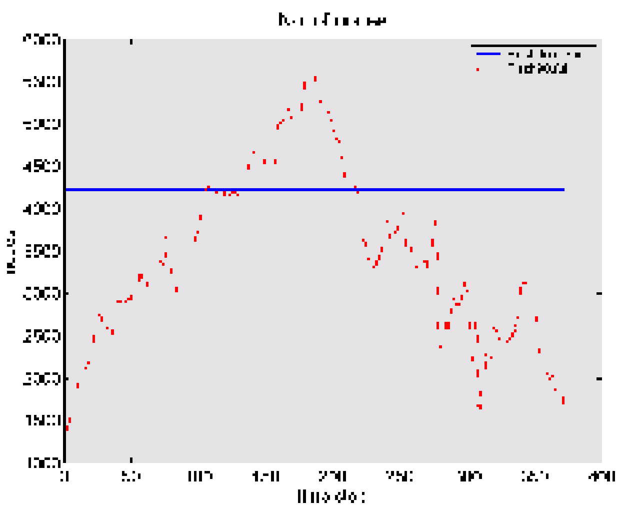
Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

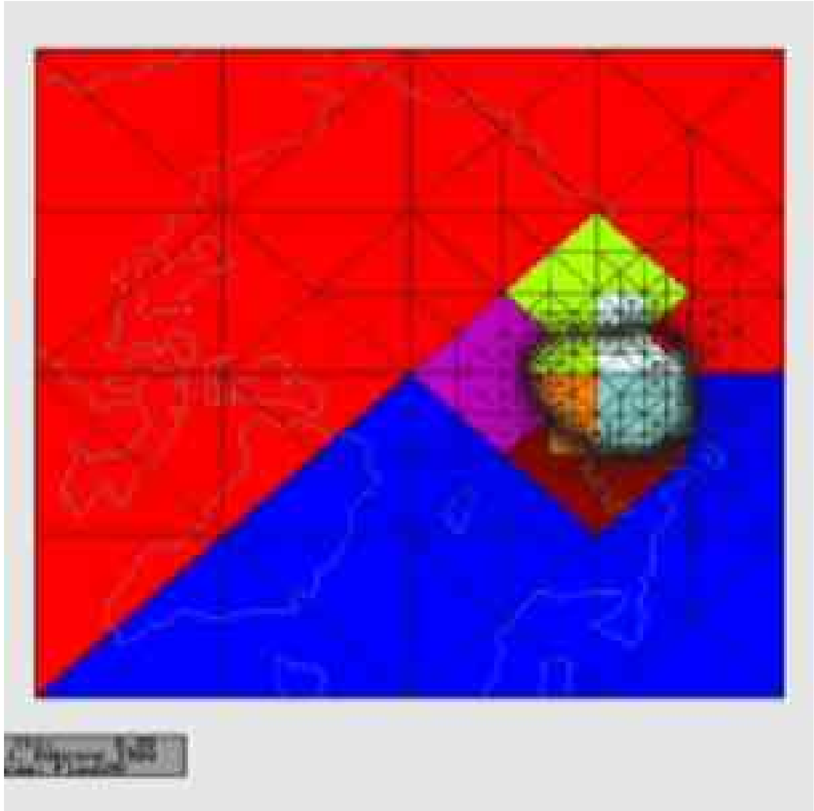
Example 1: linear advection

Simulation of tracer transport

Costs:

Uniform vs. adaptive





Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Example 2: shallow water equations

Flow over isolated mountain

$$\frac{d\zeta}{dt} + \zeta\delta + f\delta = \bar{v} \cdot \nabla_S f$$

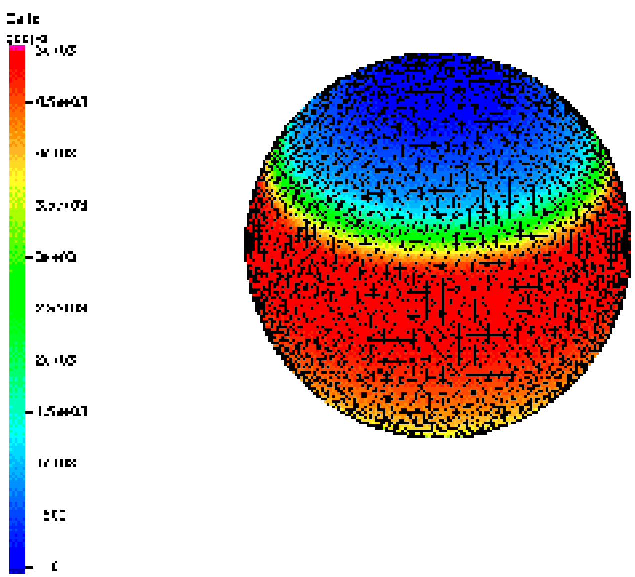
$$\frac{d\delta}{dt} + \Delta_S \Phi \quad f\zeta = (n \times v) \cdot \nabla_S f \quad J(v)$$

$$\frac{d\Phi}{dt} = (\Phi \quad \Phi_0)\delta.$$

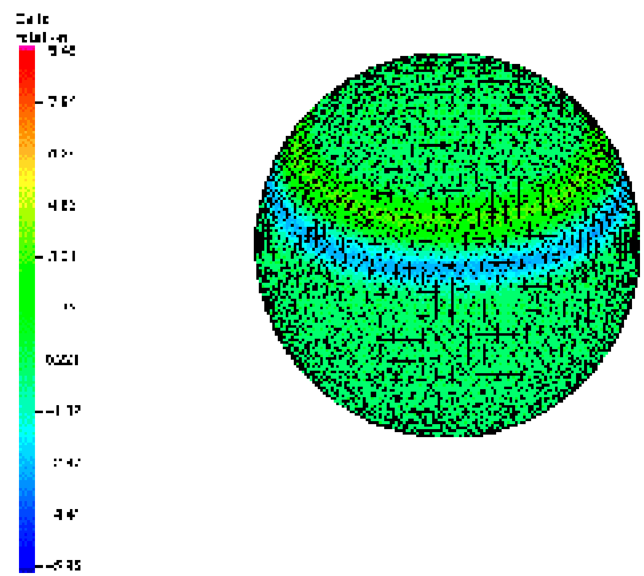
Equations in vorticity-divergence form

$$\zeta := \text{rot } \vec{V} \quad \text{(vorticity)}$$

$$\delta := \text{div } \vec{V} \quad \text{(divergence)}$$



Geopotential



Vorticity

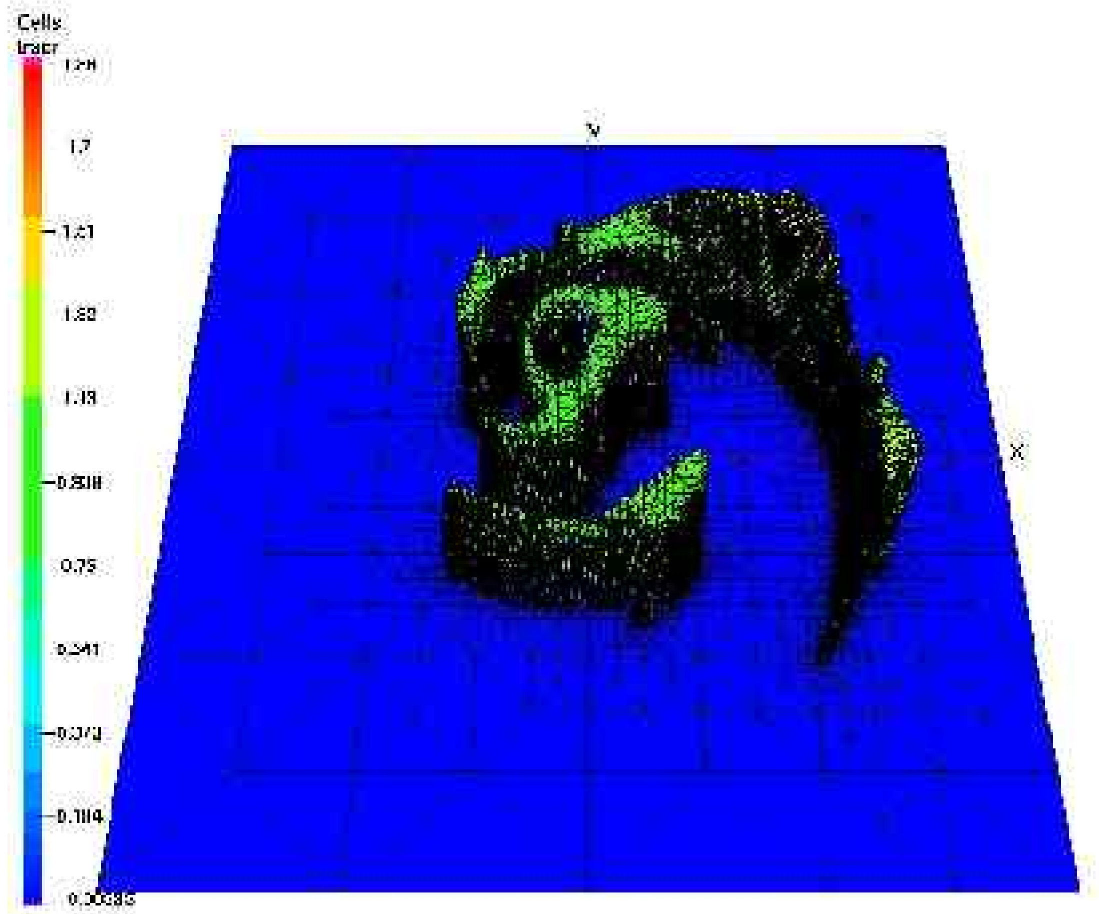
M. Läuter (2003)

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Example 3: Inverse Modeling

Problem:

Given:
- wind
- tracer density distribution



Question:

source of tracer?

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

Conclusions

- Triangular grid generation for simplicity and complex domains
- Adaptive grid refinement for accuracy and efficiency
- SFC for partitioning in parallel applications
- SFC ordering for efficient data access and matrix reordering
- Examples from tracer transport to dynamical core

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de

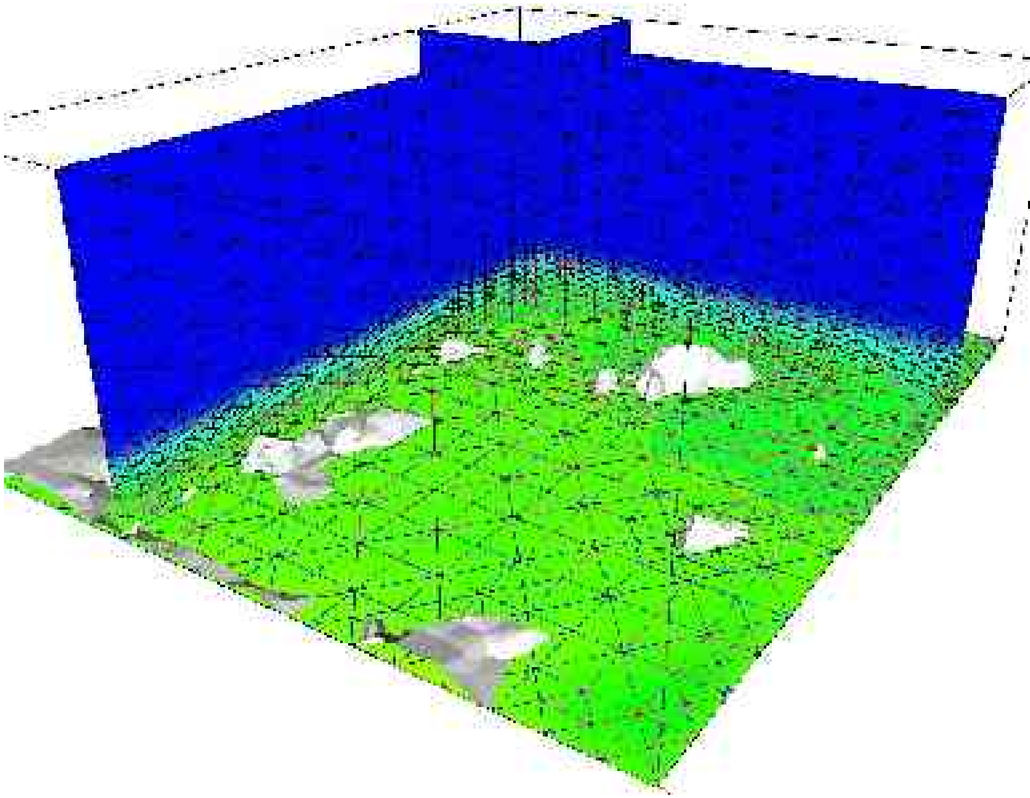
Future

Atmosphere/Ocean is 3D

Tracer transport in 3D

Wind Data from measurements

CO density from measurements (in ppb)



Future topics:

- Error estimation/ refinement criteria
- More realistic problems
- Coupling, parameterization, …

Data: E. Reimer, A. Kerschbaumer, FU Berlin

Jörn Behrens
TU München
Scientific Computing
behrens@ma.tum.de