

should be. A book like Weintraub's would go a long way toward making the subject more popular and accessible.

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**Modern Aspects of Random Matrix Theory: AMS Short Course Random Matrices, January 6–7, 2013, San Diego, California.** Edited by Van H. Vu. AMS, Providence, RI, 2014. \$56.00. viii+174 pp., hardcover. ISBN 978-0-8218-9471-2.

Random matrix theory is a beautiful mathematical topic with a rich set of deep results and difficult open problems, combining methods from—and having an impact on—many different areas, such as probability theory, combinatorics, number theory, asymptotic analysis, integrable systems, mathematical physics, linear algebra, and numerical analysis, with a wealth of stunning applications in physics, statistics, computer science, electrical engineering, and many more to name and to come. Random matrix theory has become a vast subject, as witnessed by the recent Oxford

Handbook [1] and several enormous monographs having complementary focus [2, 3, 4, 5, 6, 7], which would well make for about five thousand pages of reading. The recent thorough textbook by Anderson, Guionnet, and Zeitouni [8] and the inviting lecture notes [9] of Tao, which focuses on ideas and concepts, alleviate this rather forbidding entry for the beginner, but still, as Tropp puts it in the preface of his forthcoming book, [10], “most of the classical areas of random matrix theory remain the province of experts.” The more so for the modern aspects, the celebrated breakthroughs of the latest years. A widely accessible introductory book of reasonable length and sharp focus, striving for simplicity and understanding instead of utmost generality, is still missing but might well be too much to ask for right now.

Reading Vu's preface of the book under review made me think this little book might be up to the task, as he writes:

This volume contains surveys by leading researchers in the field, written in introductory style to quickly provide a broad picture about this fascinating and rapidly developing topic. We aim to touch most of the key points ... without putting too much technical burden on the readers. Most of the surveys are accessible with basic knowledge in probability and linear algebra.

Many readers, with already a proper exposure to some of the classical aspects of random matrices, will find the book valuable and a suitable start for pointing them to the recent research literature. However, it is not the book I hoped for, since the majority of the surveys do not really address the audience advertised in the preface.

Take, for instance, the first survey, “Lecture Notes on the Circular Law,” by Bordeave and Chafaï, and the last one, “Random Matrices: The Universality Phenomenon for Wigner Ensembles,” by Tao and Vu. They survey a celebrated sequence of recent work (by several authors) on universal limit laws under minimal assumptions, the first on the global aspects of the spectrum of non-Hermitian matrices, the second on the local aspects of Hermitian ones. Both were written long before the short course with an expert audience in mind, with lists of refer-

ences exceeding 120 entries each: the first one is an abridged (cut in half) and updated version of a paper [11] that appeared in 2012 in *Probability Surveys*, and the second one was, according to an entry on Tao's blog from Feb. 2, 2012, intended to be eventually submitted to the proceedings of a workshop on random matrix theory. The version [12] of that survey posted to the arXiv roughly a year before the short course comes with an appendix "Some Errata" reporting an issue with the *three moment theorem* that affected a series of four published papers by the authors, clearly meant to be read by the *experts*. The abstract of the Tao and Vu survey mentions a focus on their famous *four moment theorem*, which, however, makes its first appearance about 19 pages later (in section 5, the headline of which is garbled to "Extending beyond the GUE Case II. Swapping and the" with "Four Moment Theorem" apparently missing). The advertised reader with just a basic knowledge in probability and linear algebra will probably already be lost in the many technical details, detours, and ramifications long before that.

There are three more surveys to be found in the book. One, "Free Probability and Random Matrices," by Guionnet, aims to give a very short introduction to a beautiful subject, noncommutative moment-based probability with the algebraic concept of free independence, in just about a dozen pages. It is far too compressed to be digestible for the uninitiated and probably does not give much new insight for the adept. This is in sharp contrast to the well-crafted chapter on free probability theory by the same author in [8], which is eight times as long. Compare the readability of the proofs that independent Wigner matrices are asymptotically free (Theorem 2.1 here and Theorem 5.4.2 in [8]): without any mention of the moment-based proof of Wigner's semicircle law, the proof has lost its base. This survey finishes with a section on Brown measures and the celebrated *single ring theorem* of the author. The connection to free probability ("the Brown measure can be explicitly computed by free probability tools") remains a mystery to the nonexpert, though.

The next one, "Random Matrix Theory, Numerical Computations and Appli-

cations," by Edelman, Sutton, and Wang, is centered on the intriguing thesis that "a computational trick can open entirely new approaches in theory." Though giving a compelling example of that thesis by obtaining random differential operator representations of limit spectral distributions by first reducing the eigenvalue problem for random matrices to tridiagonal models and then reinterpreting these models as discretizations of second order differential operators, it remains a mixed bag. It spends much space on introducing the Itô integral (but calling it "white noise transformation") on the one hand, but gives only short shrift to aspects from numerical linear algebra ("Can we histogram without histogramming? The answer is Yes!"), especially in the subsection on superfast computation, and to the relation between Sturm sequences, shooting, and Sturm–Liouville theory. The speculative section on "Ghosts and Shadows" is a story covert in darkness and fog: asking the reader to "make great strides by letting go of what at first seems so dear" addresses, if anyone concrete, the fellow expert. Anyone else would not understand why the authors talk about "ghost Jacobian computations" in *fractional* dimensions without being shown the role of such calculations in, say, the classical transformation to tridiagonal models first. The short subsection on "Jack Polynomials and Ghosts" exemplifies the shortcomings of this book: it first tells the reader that Jack introduced his polynomials in 1970 and that they "are closely related to our ghosts," and next continues by mentioning that "with MOPS, we can press a few buttons before understanding the polynomials just to see what they look like" illustrated by three concrete cases, then relates them in two lines to Schur polynomials and zonal polynomials but, finally, "we will not define the Jack polynomials here." After this refusal, the puzzled reader, who is still wondering what all these polynomials and MOPS are, is surprised to learn that "we expect that the Jack polynomial formula gives consistent moments" and stumbles about some formula coming out of the blue which is said to be an analogue to one on page 243 of a book by Muirhead. It is difficult not to make a satire out of this, but there is the question: who is expected to profit

from reading such subsections (and there are many of those to be found in the book)?

The last survey to be discussed, “Recent Developments in Non-asymptotic Theory of Random Matrices,” by Mark Rudelson, investigates spectral properties, such as bounds on the smallest singular value which are valid with high probability for matrices of a large, but fixed size: material which is of great interest, besides for random matrix theory proper, for topics such as compressive sensing and smoothed analysis of algorithms. To my impression it is the most balanced and readable survey in the book, with careful exposition of the basic ideas and methods and well chosen decisions of when to refer to the literature. The sharpness of the estimates requires still a considerable amount of technical preparation and detail, however.

Interestingly, though all the other surveys refer to the work of Rudelson and Vershynin, there is no reference to the corresponding survey of Rudelson in this book. Likewise, Hermitization is used both in the surveys of Bordenave and Chafaï as well as the one of Guionnet. Cross-referencing of such and other material could have made a more coherent experience for the (expert) reader. The publisher has supplied a common index for the book, which is laudable and, together with the comprehensive list of references given in each of the surveys, increases the usefulness of the book for readers who are looking for a starting point in the research literature. But an introduction to the modern aspects that really takes care of those readers with just a basic knowledge in probability theory and linear algebra, *random matrices for the people*, has yet to be written. In the meantime, this book is a compendium of some beautiful and important work by a collection of outstanding mathematicians.

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**Solving Transcendental Equations: The Chebyshev Polynomial Proxy and Other Numerical Rootfinders, Perturbation Series, and Oracles.** By John P. Boyd. SIAM, Philadelphia, 2014. \$99.00. xviii+460 pp., soft-cover. ISBN 978-1-611973-51-8.

I reviewed a prepublication draft of this book (hereinafter STE) for the SIAM editors and suggested significant changes before publication. I “pulled no punches” but did recommend publication once some issues were addressed. They were, of course, as were those of other reviewers.

STE as published is quite different from that earlier draft, and I am very happy to have this chance to review the finished product as well. STE is very much worth reading twice anyway, containing as it does many