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A Perplexing Polynomial Puzzle, Revisited

From Folkmar Bornemann (Technical University of Munich, Munich, Germany; bornemann@ma.tum.de) and Stan Wagon (Macalester College, St. Paul, MN; wagon@macalester.edu):

A problem in the March 2005 issue (pages 100 and 159) tells how one can determine a polynomial $P(x)$ with nonnegative integer coefficients from just two of its values. In the solution, only values at integers are used. In fact, as we show below, any such polynomial can be determined from finitely many digits of just *one* of its values!

Suppose that Alice chooses a polynomial $P(x)$ whose coefficients are nonnegative integers, and Bob wants to discover it. He asks for the decimal digits of $P(\pi)$. As soon as he hears "point," he asks Alice to pause. Letting K denote this integer, $\lfloor P(\pi) \rfloor$, he computes $d = \lfloor \log_3 K \rfloor$. Then every coefficient of $P(x)$ must be between 0 and K (since $K \geq P(1)$) and the degree of $P(x)$ is at most d (since $K \geq P(3) \geq 3^d$).

Bob then makes a list S (which will be finite) of all those polynomials $Q(x)$ of degree d or less whose coefficients are nonnegative integers at most K. He chooses a starting precision N and computes $Q(\pi)$ to N decimal places for each polynomial $Q(x)$ in S. If the results are not all different, by repeatedly taking N to be twice as large as before, Bob eventually succeeds in getting all of the $Q(\pi)$ computed to N decimal places to be different. This works because π is transcendental. At this point, he asks Alice for $P(\pi)$ to N places and finds which one of his $Q(\pi)$ values agrees with this. This $Q(x)$ must be $P(x)$!

