ERRATUM

FOLKMAR BORNEMANN

As pointed out by Georg Wechslberger, there is an asymptotic error term $O(\|E\|^2_{\mathcal{J}_1})$ missing from the formulation and proof of Lemma 4.1 of my paper

> Bornemann, F.: 2010, On the numerical evaluation of Fredholm determinants, *Math. Comp.* **79**, 871–915.

This has no consequence to the substance of the lemma and the applications made of it in the paper, whatsoever. In particular, the important assertion $\kappa_{\mathrm{abs}} \leqslant 1$ remains correct. The lemma and its proof should read as follows (corrections marked in red):

Lemma 4.1. *Let $A \in \mathcal{J}_1(\mathcal{H})$ be selfadjoint, positive-semidefinite with $\lambda_1(A) < 1$. Then, as $\|E\|_{\mathcal{J}_1} \to 0$,*

$$|\det(I - (A+E)) - \det(I - A)| \leqslant \|E\|_{\mathcal{J}_1} + O(\|E\|^2_{\mathcal{J}_1}). \tag{4.1}$$

That is, the condition number κ_{abs} of the determinant $\det(I - A)$, with respect to absolute errors measured in trace class norm, is bounded by $\kappa_{abs} \leqslant 1$.

Proof. Because of $1 > \lambda_1(A) \geqslant \lambda_2(A) \geqslant \cdots \geqslant 0$, the inverse operator $(I - A)^{-1}$ exists. The product formula (3.1) implies $\det(I - A) > 0$; the multiplicativity (3.5) of the determinant gives

$$\det(I - (A+E)) = \det(I - A)\det(I - (I - A)^{-1}E).$$

Upon applying Plemelj's formula (3.3) and the estimates (2.3) and (2.5) we get

$$|\log\det(I - (I - A)^{-1}E)| = \left|\operatorname{tr}\left(\sum_{n=1}^{\infty} \frac{((I - A)^{-1}E)^n}{n}\right)\right|$$

$$\leqslant \sum_{n=1}^{\infty} \frac{\|(I - A)^{-1}\|^n \cdot \|E\|^n_{\mathcal{J}_1}}{n} = \log\left(\frac{1}{1 - \|(I - A)^{-1}\| \cdot \|E\|_{\mathcal{J}_1}}\right)$$

if $\|(I - A)^{-1}\| \cdot \|E\|_{\mathcal{J}_1} < 1$. Hence, exponentiation yields

$$1 - \|(I - A)^{-1}\| \cdot \|E\|_{\mathcal{J}_1} \leqslant \det(I - (I - A)^{-1}E)$$

$$\leqslant \frac{1}{1 - \|(I - A)^{-1}\| \cdot \|E\|_{\mathcal{J}_1}} \leqslant 1 + \|(I - A)^{-1}\| \cdot \|E\|_{\mathcal{J}_1} + O(\|E\|^2_{\mathcal{J}_1}),$$

that is

$$|\det(I - (A+E)) - \det(I - A)| \leqslant \det(I - A) \cdot \|(I - A)^{-1}\| \cdot \|E\|_{\mathcal{J}_1} + O(\|E\|^2_{\mathcal{J}_1}).$$

Now, by the spectral theorem for bounded selfadjoint operators we have

$$\|(I - A)^{-1}\| = \frac{1}{1 - \lambda_1(A)} \leqslant \prod_{n=1}^{N(A)} \frac{1}{1 - \lambda_n(A)} = \frac{1}{\det(I - A)}$$

and therefore $\det(I - A) \cdot \|(I - A)^{-1}\| \leqslant 1$, which finally proves the assertion. □

Date: February 8, 2010; *E-mail address*: `bornemann@ma.tum.de`.